Uncertainty, Financial Frictions, and Investment Dynamics

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September 28, 2010

Abstract

This paper analyzes—both empirically and theoretically—how fluctuations in uncertainty interact with financial market imperfections in determining economic outcomes. In a standard bond-contracting framework, an increase in uncertainty benefits equity holders at the expense of bondholders, and to the extent that firms face significant frictions in financial markets, increased uncertainty implies a higher cost of capital and hence a decline in investment. The reduction in credit supply also hampers the efficient reallocation of capital and causes an endogenous decline in total factor productivity (TFP) that amplifies the economic downturn. Using both aggregate time-series and firm-level data, we find strong evidence supporting the notion that financial frictions play a major role in shaping the uncertainty-investment nexus. We then develop a tractable general equilibrium model in which individual firms face time-varying uncertainty and imperfect capital markets when issuing risky bonds and equity to finance investment projects. We calibrate the uncertainty process using micro-level estimates of shocks to the firms' profits and show that the combination of uncertainty shocks and financial frictions can generate fluctuations in economic activity that are observationally equivalent to the TFP-driven business cycles.

JEL Classification: E22, E32, G31
Keywords: uncertainty, financial frictions, investment, capital reallocation, TFP

We appreciate comments and suggestions from Jeff Campbell, Nobuhiro Kiyotaki, Neng Wang, and the seminar participants at the Federal Reserve Board, the Federal Reserve Banks of Dallas and Chicago, the 2009 LAEF Conference at the UC–Santa Barbara, the 2009 Macro System Conference at the Federal Reserve Bank of San Francisco, the 2010 Winter Meetings of the Econometric Society, Princeton University, the 2010 Meeting of the Society for Economic Dynamics, and the 2010 NBER Summer Institute. Robert Kurtzman and Oren Ziv provided outstanding research assistance. All errors and omissions are our own responsibility alone. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of anyone else associated with the Federal Reserve System.

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1 Introduction

The countercyclical behavior of the cross-sectional dispersion of economic returns such as labor income, business cash flows, and equity valuations is one of the stylized facts of cyclical fluctuations.[1] In macroeconomics, irreversible investment provides the traditional mechanism through which changes in uncertainty affect economic activity (Bernanke [1983], Dixit and Pindyck [1994], Caballero and Pindyck [1996], and more recently, Bloom [2009] and Bloom et al. [2009]).[2] Financial market imperfections provide an alternative, though not necessarily an exclusive, channel through which fluctuations in uncertainty can affect economic activity. In the standard framework used to price corporate debt (Merton [1974]), the payoff structure of levered equity—under limited liability—resembles the payoff of a call option, while the bondholders face the payoff structure that is equivalent to that of an investor writing a put option. An increase in the riskiness of the firm's assets thus benefits equity holders at the expense of bondholders, implying a rise in the default-risk premium to compensate bondholders for increased uncertainty. To the extent that default is costly and external funds command a premium, an increase in uncertainty raises the costs of capital and causes a decline in investment spending.

The aim of this paper is to investigate—both empirically and theoretically—the relationship between uncertainty and investment in the context of imperfect financial markets. First, we construct a proxy for idiosyncratic time-varying economic uncertainty using daily firm-level stock returns for the U.S. nonfinancial corporate sector. We use this uncertainty measure to examine the dynamic interaction between output, investment, uncertainty, and credit spreads on corporate bonds—an indicator of financial stress—within a structural vector autoregression (VAR) framework. Our empirical results indicate that conditions in the corporate debt markets are an important conduit through which fluctuations in uncertainty are propagated to the real economy. Unanticipated increases in uncertainty lead to a significant widening of credit spreads, a drop in output, and a protracted decline in business fixed investment.

We complement this analysis by constructing a new firm-level panel data set that combines information on prices of individual corporate bonds trading in the secondary market with our estimates of firm-specific uncertainty and the issuers' income and balance sheet information. Results from this panel-data analysis confirm the aggregate time-series findings: Conditional on the firm's leverage, profitability, and other indicators of credit quality, our firm-specific measure of uncertainty is an important determinant—both economically and

[1] See, for example, Campbell and Taksler [2003], Storesletten et al. [2004] Eisfeldt and Rampini [2006], and Bloom et al. [2009].

[2] As shown by Abel [1983], Veracierto [2002], and Bachmann and Bayer [2009], the effect of uncertainty on investment in the presence of irreversibilities can be theoretically ambiguous, and it depends importantly on the assumptions regarding the initial accumulation of capital, market structure, and the equilibrium setting.

statistically—of credit spreads on the firm's outstanding bonds. According to our results, an increase in uncertainty of 10 percentage points boosts credit spreads about 15 basis points, an economically substantial effect given the extent of the observed variation in idiosyncratic uncertainty.

We also find that conditional on investment fundamentals—that is, proxies for the marginal product of capital—the long-run elasticity of investment demand with respect to uncertainty lies in the range between -0.70 and -0.40, implying that a 10 percentage point increase in idiosyncratic uncertainty leads to a decline in the investment rate between 2.0 and 3.5 percentage points. However, once the information content of credit spreads is taken into account, the impact of uncertainty on investment ceases to be statistically or economically significant. Capital formation, in contrast, remains highly sensitive to the firm-specific financial conditions, with a 100 basis points rise in credit spreads leading to a drop in the investment rate of more than a full percentage point. All told, these aggregate and firm-level result strongly support the notion that the impact of uncertainty on investment is influenced to a significant degree by the presence of financial market frictions.

To provide a theoretical context for our empirical findings, we construct a tractable bond-contracting model of the type analyzed by Bernanke et al. [1999], Cooley and Quadrini [2001], Hennessy and Whited [2007], and Philippon [2009]. We embed this contracting framework into a standard capital accumulation problem, in which firms employ a production technology that is subject to a persistent idiosyncratic shock, the variance of which is allowed to vary over time according to a stochastic law of motion. The firms make investment decisions subject to a full range of choices regarding their capital structure—internal funds, debt, and equity financing—in an environment where external funds are costly because of frictions in financial markets.[3]

The model simulations accord well with our empirical results along a number of dimension. An increase in uncertainty causes corporate bond prices to fall and credit spreads to widen immediately as investors demand greater protection against the increased downside risk. The rise in private yields pushes up the effective cost of capital, because the firms cannot costlessly replace debt with new equity to finance their investment projects. As a result, aggregate investment falls in response to an increase in uncertainty. Similar to the financial accelerator mechanisms emphasized by Kiyotaki and Moore [1997], Bernanke et al. [1999], Christiano et al. [2009], and Hall [2010], the effect of uncertainty on economic activity is amplified through the endogenous movements in the price of assets that serve as

[3]Cooley and Quadrini [2001], Hennessy and Whited [2007], and Philippon [2009] consider similar contracting frameworks, though only in partial equilibrium. Bernanke et al. [1999] do allow for general equilibrium feedback effects but consider only debt financing. In our setting, the combination of persistent idiosyncratic productivity shocks and a debt-renegotiation problem delivers a considerably richer set of dynamic implications, because the joint distribution of productivity shocks and the condition of the firms' balance sheets become the state variables of the model.

collateral for future borrowing.

In contrast to the aforementioned literature, our framework allows for a meaningful degree of heterogeneity in the technology and financial conditions of firms in the economy. In such an environment, an increase in uncertainty impedes the amount of reallocation of factor inputs from less productive firms with high net worth to more productive firms with low net worth. As a result, fluctuations in uncertainty cause movements in aggregate TFP, which further amplify the effect of uncertainty on economic activity. This mechanism implies that capital reallocation in our model is countercyclical, consistent with the evidence reported by Eisfeldt and Rampini [2006], who motivate the procyclical nature of capital reallocation by assuming countercyclical capital adjustment costs. In our model, by contrast, the presence of financial market frictions generates an endogenous increase in the cost of reallocation during the uncertainty-induced economic downturns, leading to the type of productivity dynamics emphasized by Kiyotaki [1998].[4]

The remainder of the paper is organized as follows. Section 2 contains our empirical findings; the first part focuses on the aggregate time-series evidence regarding the role that credit spreads play in shaping the dynamic response of the economy to fluctuations in uncertainty; the second part buttresses our time-series results with new firm-level evidence on the link between uncertainty, credit spreads, and capital formation. The theoretical model is presented in Section 3. Section 4 describes the calibration of the model's parameters, while Section 5 presents our simulation exercises and discusses the model's implications against the background of our earlier empirical findings. Section 6 concludes.

2 Empirical Evidence

2.1 Measuring Time-Varying Economic Uncertainty

We utilize high-frequency (i.e., daily) firm-level equity returns to construct our benchmark estimate of time-varying economic uncertainty. The advantage of using equity valuations to measure uncertainty is that asset prices should, in principle, encompass all aspects of the firm's environment that investors view as important. Specifically, from the Center for Research in Security Prices (CRSP) data base, we extracted daily stock returns for all U.S. nonfinancial corporations with at least 1,250 trading days (essentially five years) of data. This selection criterion yielded a panel of 10,729 firms over the period from July 1, 1963 (1963:Q3) to December 31, 2009 (2009:Q4).[5]

[4]More recently, the role of resource misallocation in shaping productivity dynamics has been analyzed by Kleenow and Hsieh [2009] and Basu et al. [2009].

[5]To ensure that our results were not driven by a small number of extreme observations, we eliminated all observations with a daily absolute return in excess of 100 percent.

Our benchmark estimate of uncertainty is based on the following three-step procedure. In the first step, we remove the systematic component of (excess) equity returns using the standard Fama and French [1992] 3-factor model:

$$(r_{it_n} - r^f_{t_n}) = \alpha_i + \beta^M_i (r^M_{t_n} - r^f_{t_n}) + \beta^{SMB}_i SMB_{t_n} + \beta^{HML}_i HML_{t_n} + u_{it_n}, \tag{1}$$

where i indexes firms and t_n, $n = 1, \ldots, N$, indexes trading days in quarter t. In equation (1), r_{it_n} denotes the (total) log return of firm i; $r^f_{t_n}$ is the continuously-compounded 3-month Treasury yield (i.e., the risk-free rate); $r^M_{t_n}$ is the value-weighted (total) log return for the market as a whole; and SMB_{t_n} and HML_{t_n} are the Fama-French "risk" factors.

In the second step, we calculate the quarterly standard deviation of daily idiosyncratic returns for each firm i:

$$\sigma_{it} = \sqrt{\frac{1}{N-1} \sum_{n=1}^{N} (\hat{u}_{it_n} - \hat{\bar{u}}_{it})^2}, \tag{2}$$

where $\hat{u}_{it_n}$ denotes the OLS residual from equation (1) and $\hat{\bar{u}}_{it} = N^{-1} \sum_{n=1}^{N} \hat{u}_{it_n}$ is the sample mean of daily idiosyncratic returns in quarter t. Thus, σ_{it} is an estimate of time-varying equity volatility for firm i, a measure that abstracts from the common risk factors that drive differences in expected returns across firms.

In the final step, we assume that the firm-specific uncertainty in equation (2) follows an autoregressive process of the form:

$$\log \sigma_{it} = \gamma_i + \delta_i t + \rho \log \sigma_{i,t-1} + v_t + \epsilon_{it}, \quad |\rho| < 1 \text{ and } \epsilon_{it} \sim N(0, \omega^2); \tag{3}$$

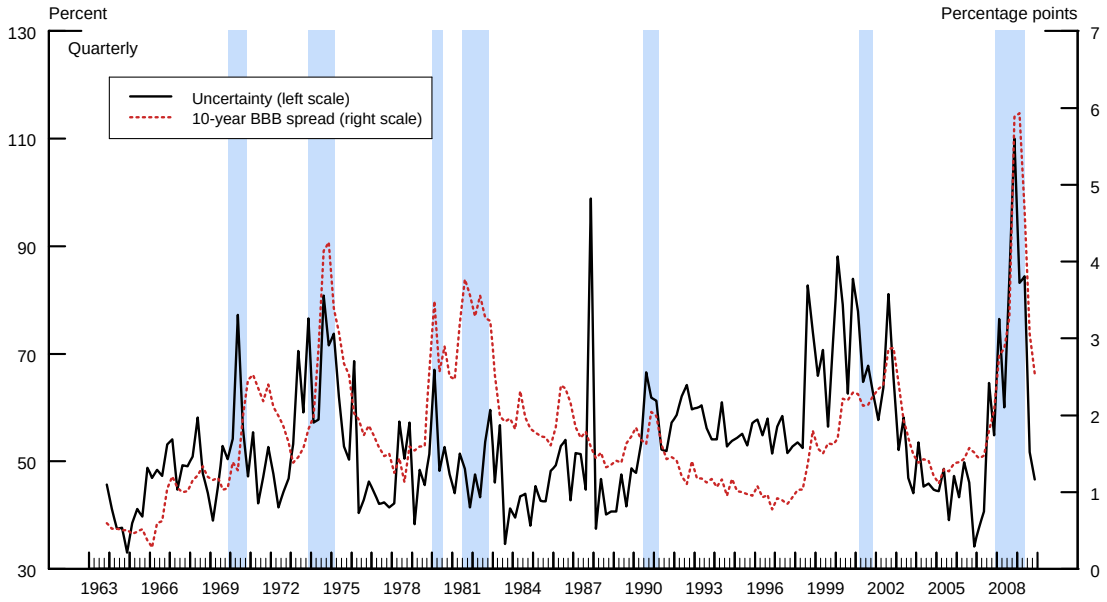
where γ_i denotes a firm fixed effect intended to control for the cross-sectional heterogeneity in σ_{it}, whereas the firm-specific term $\delta_i t$ captures the secular upward trend in the idiosyncratic risk of publicly-traded U.S. firms documented by Campbell et al. [2001]. Our benchmark estimate of time-varying macroeconomic uncertainty is the sequence of time fixed effects v_t, $t = 1, \ldots, T$, which captures shocks to idiosyncratic volatility that are common to all firms. We estimate the parameters of equation (3) by OLS, which yields an estimate of $\rho = 0.423$, an indication that idiosyncratic equity volatility tends to be fairly persistent.[6] The specification also fits the data quite well, explaining almost 75 percent of the variation in the dependent variable.

Figure 1 shows our benchmark estimate of time-varying uncertainty derived from the estimated time fixed effects in equation (3).[7] The figure also plots the spread between the

[6] Because the average firm is in the panel for almost 60 quarters, the bias of the OLS estimator, owing to the presence of a lagged dependent variable and firm fixed effects, is negligible (see, for example, Arellano [2003]).

[7] To ease the interpretation, the estimates of v_t have been re-scaled and expressed in annualized percent.

Figure 1: Uncertainty and Corporate Credit Spreads



NOTE: Sample period: 1963:Q4–2009:Q4. The solid line depicts our benchmark estimate of time-varying uncertainty based on equity valuations (see text for details). The dotted line depicts the spread between the 10-year yield on BBB-rated corporate bonds and the 10-year Treasury yield. The shaded vertical bars denote NBER-dated recessions.

10-year yield on BBB-rated corporate bonds and the 10-year Treasury yield, an indicator of conditions in the corporate debt markets. Clearly evident is the fact that both series are countercyclical, typically rising sharply before recessions.

2.2 Aggregate Time-Series Evidence

In this section, we use a VAR framework to investigate the interaction between our benchmark estimate of economic uncertainty, business financial conditions, and real economic activity. In particular, we estimate a VAR consisting of the following six endogenous variables: the logarithm of real GDP (y_t); the logarithm of real business fixed investment (i_t); the logarithm of the GDP price deflator (p_t); the (nominal) effective federal funds rate (f_t) as an indicator of the stance of monetary policy; the 10-year BBB-Treasury credit spread (s_t); and our benchmark estimate of time-varying uncertainty (v_t). The VAR is estimated over the 1963:Q3–2009:Q4 period using four lags of each endogenous variable and, in addition to a constant term, also includes dummy variables for 1987:Q4 and 2008:Q4 as two additional exogenous regressors.[8]

[8]The inclusion of these two dummy variables is motivated by the fact that the volatility spike in 1987:Q4 and the surge in uncertainty and credit spreads during the period of acute financial turmoil in late 2008

We focus on the implications of uncertainty shocks on credit spreads and economic activity. To identify these disturbances, we employ a standard recursive ordering technique, in which shocks to uncertainty have an immediate impact on credit spreads and short-term interest rates, but they affect the output, investment, and prices with a lag. To provide a point of comparison, we rely on the same recursive ordering to examine the impact of shocks to credit spreads—that is, "financial shocks"—that are orthogonal to contemporaneous movements in uncertainty. We also consider a specification that reverses this causal ordering, which allows to examine the implications of uncertainty shocks conditional on the information contained in the current level of credit spreads.

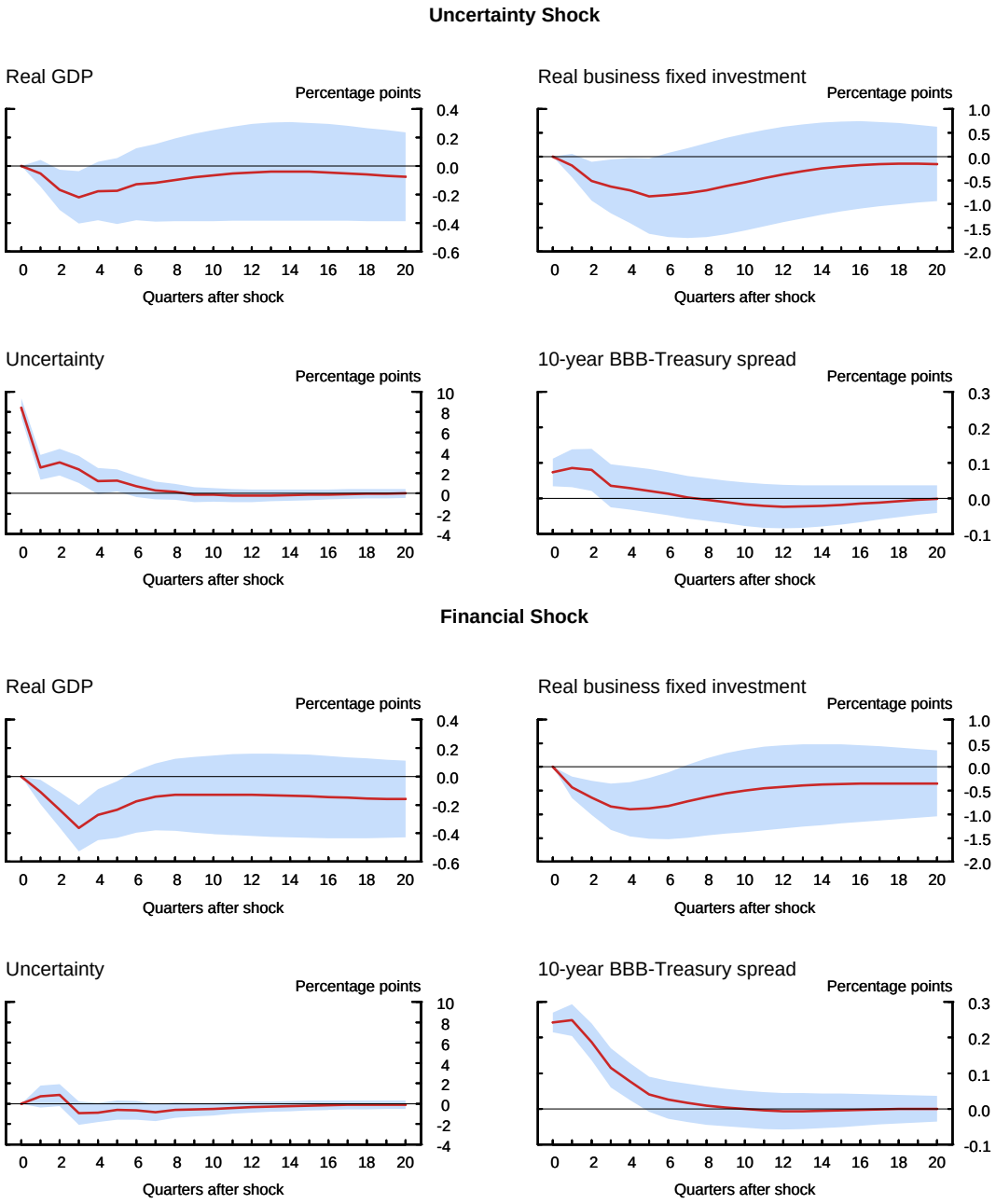
Figure 2 plots the impulse response functions of selected variables to uncertainty and financial shocks orthogonalized using the baseline identification scheme. Given these identifying assumptions, an unanticipated increase in uncertainty causes an immediate widening of corporate bond spreads. Moreover, this uncertainty shock has significant adverse consequences for economic activity. Output declines almost immediately, reaching a trough about a year after the initial spike in uncertainty. The response of investment is considerably more pronounced and protracted, as capital spending falls steadily, bottoming out almost a full percentage point below the trend five quarters after the shock. A financial shock, which causes an increase of about 25 basis points in the 10-year BBB-Treasury spread, similarly leads to a significant contraction in economic activity. The shock to credit spreads, however, has no discernible effect on uncertainty.

Figure 3 shows the implications of uncertainty and financial shocks orthogonalized using an alternative scheme in which credit spreads are ordered before uncertainty. Under these identifying assumptions, an unanticipated increase in uncertainty has no statistically discernible effect on the real economy. Financial shocks, in contrast, have significant and long-lasting effects on both output and investment. A one standard deviation shock to the 10-year BBB-Treasury spread is associated with an immediate jump in uncertainty, a substantial decline in real GDP, and a protracted fall in business fixed investment. Indeed, the magnitude and the shape of the impulse response functions of both output and investment are very similar to those shown in Figure 2.

In summary, the time-series evidence presented above implies that an increase in uncertainty leads to an economically and statistically significant widening of credit spreads on corporate bonds, a drop in output, and a protracted decline in business fixed investment. The evidence also suggests that changes in credit conditions are an important part of the

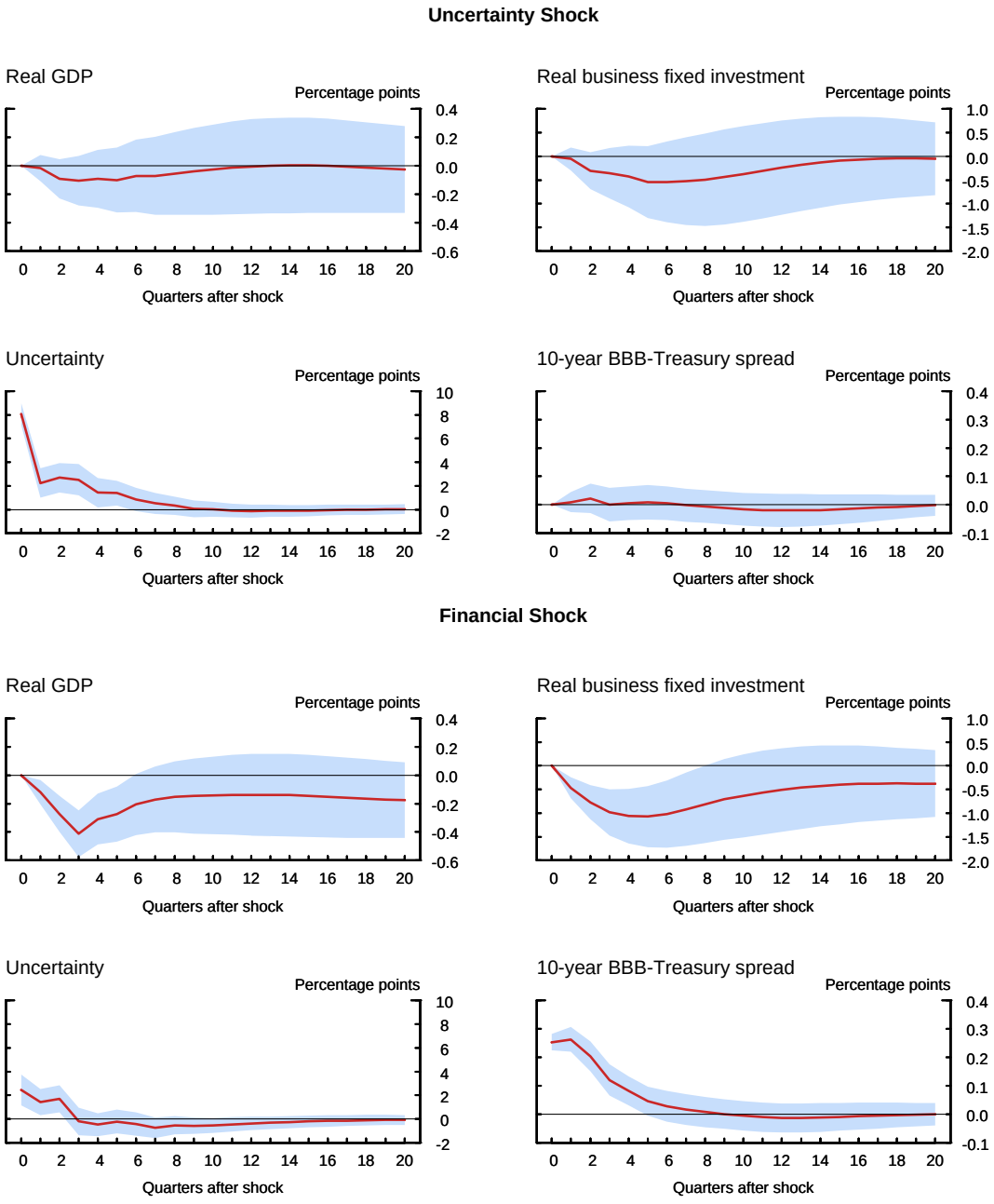
appear to be well outside historical norms. Indeed, standard regression diagnostics indicate that these two observations exert an unduly large influence on the estimated coefficients, especially in the uncertainty and credit spread equations. By including these two dummy variables in the VAR, we ensure that our results are not driven by a small number of extreme observations. Nonetheless, the results reported in the paper are robust to the exclusion of these two dummies from the VAR.

Figure 2: Dynamic Implications of Uncertainty and Financial Shocks
(Baseline Identification Scheme)



NOTE: The top four panels depict the impulse response functions to an orthogonalized one standard deviation shock to our benchmark estimate of time-varying uncertainty. The bottom four panels depict the impulse response functions to an orthogonalized one standard deviation shock to the 10-year BBB-Treasury spread. The baseline identification scheme corresponds to the following recursive ordering of the VAR system: $(y_t, i_t, p_t, v_t, s_t, f_t)$. Shaded bands represent 95-percent confidence intervals based on 2,000 bootstrap replications.

Figure 3: Dynamic Implications of Uncertainty and Financial Shocks
(Alternative Identification Scheme)



NOTE: The top four panels depict the impulse response functions to an orthogonalized one standard deviation shock to our benchmark estimate of time-varying uncertainty. The bottom four panels depict the impulse response functions to an orthogonalized one standard deviation shock to the 10-year BBB-Treasury spread. The alternative identification scheme corresponds to the following recursive ordering of the VAR system: $(y_t, i_t, p_t, s_t, v_t, f_t)$. Shaded bands represent 95-percent confidence intervals based on 2,000 bootstrap replications.

transmission mechanism propagating uncertainty shocks to the real economy. Indeed, our results indicate that once shocks to uncertainty are orthogonalized with respect to the contemporaneous information from the corporate bond market, fluctuations in uncertainty have no statistically significant effect on economic activity.

2.3 Firm-Level Evidence

In this section, we utilize a new firm-level data set to provide additional evidence regarding the role of financial market frictions as a determinant of investment dynamics in response to fluctuations in economic uncertainty. Following Leahy and Whited [1996], our empirical strategy involves regressing investment on the firm-specific estimate of idiosyncratic uncertainty, while controlling for the fundamental determinants of investment spending.

Given the focus on the interaction between uncertainty and financial frictions, our regression specification also includes credit spreads at the level of an *individual* firm. To that purpose, we constructed a panel data set of almost 1,000 publicly-traded U.S. nonfinancial firms covered by CRSP and S&P's Compustat over the 1973–2009 period. The distinguishing characteristic of these large corporations is that a significant portion of their outstanding liabilities is in the form of long-term bonds that are actively traded in the secondary market. We use the secondary market prices of individual securities to construct firm-level credit spreads, which are then matched to the issuer's income and balance sheet data. (The description of the bond-level data set and the details regarding the construction of credit spreads are contained in Appendix A.)

2.3.1 Uncertainty, Credit Spreads, and Investment

The first empirical exercise using our firm-level data examines the link between credit spreads and uncertainty. We estimate the following (reduced-form) bond-pricing equation:

$$\log s_{it}[k] = \beta_1 \log \sigma_{it} + \beta_2 R^E_{it} + \beta_3 [\Pi/A]_{it} + \beta_4 \log[D/E]_{i,t-1} + \theta' \mathbf{x}_{it}[k] + \epsilon_{it}[k], \qquad (4)$$

where $\log s_{it}[k]$ denotes the credit spread of a bond issue k in period t, a security that is a liability of firm i.[9] In addition to our estimate of idiosyncratic uncertainty σ_{it} (see equation (2)), credit spreads are allowed to depend on the firm's repayment prospects, as measured by the firm's realized quarterly return on equity R^E_{it} and the ratio of operating income to assets $[\Pi/A]_{it}$, while the ratio of the book value of total liabilities to the market value of the firm's equity—denoted by $[D/E]_{it}$—captures the strength of the firm's balance

[9] Although our data on credit spreads are at a monthly frequency, the requisite income and balance sheet information from Compustat is available only at a quarterly frequency. In addition, the firms' fiscal years/quarters end at different months of the year. The timing of our firm-level data reflects these differences as our observations occur at different months but are spaced at regular quarterly (i.e., three-month) intervals.

Table 1: Uncertainty and Credit Spreads

Explanatory Variable	(1)	(2)	(3)	(4)
$\log \sigma_{it}$	0.876	0.594	0.616	0.238
	(0.027)	(0.018)	(0.018)	(0.019)
R^E_{it}	-0.351	-0.441	-0.430	-0.233
	(0.033)	(0.023)	(0.023)	(0.016)
$[\Pi/A]_{it}$	-4.598	-2.222	-1.927	-0.915
	(0.939)	(0.558)	(0.511)	(0.432)
$\log[D/E]_{i,t-1}$	0.223	0.063	0.067	0.123
	(0.043)	(0.020)	(0.018)	(0.015)
Adj. R^2	0.472	0.621	0.629	0.785
Credit Rating Effects[a]	-	(0.000)	(0.000)	(0.000)
Industry Effects[b]	-	-	(0.000)	(0.000)
Time Effects[c]	-	-	-	(0.000)

NOTE: Sample period: bond-level monthly data from January 1973 to December 2009 at a quarterly frequency (No. of firms/bonds = 944/5072; Obs. = 88,447). Dependent variable is $\log(s_{it}[k])$, the logarithm of the credit spread of bond k in month t (issued by firm i). All specifications include a constant, a vector of control variables $\mathbf{x}_{it}[k]$ (not reported) and are estimated by OLS. Heteroscedasticity- and autocorrelation-consistent asymptotic standard errors are clustered at the firm level and are reported in parentheses.

[a] p-value for the test of the null hypothesis of the absence of fixed credit rating effects.

[b] p-value for the test of the null hypothesis of the absence of fixed industry effects.

[c] p-value for the test of the null hypothesis of the absence of time fixed effects.

sheet. The vector $\mathbf{x}_{it}[k]$ contains variables capturing bond- or firm-specific characteristics that could influence bond yields through either liquidity or term premiums, including the bond's duration, the amount outstanding, the bond's (fixed) coupon rate, and an indicator variable that equals one if the bond is callable and zero otherwise.[10]

Table 1 contains these estimation results. According to column 1, an increase in uncertainty leads to a significant widening of credit spreads—the elasticity estimate of 0.843 implies that an increase in uncertainty of 10 percentage points in quarter t will boost credit spreads more than 50 basis points. The coefficients on the remaining key variables are also economically and statistically highly significant and have their expected signs: Strong profitability performance, as evidenced by a high realized return on equity or an increase in the ratio of operating income to assets, is associated with a narrowing of credit spreads, whereas an increase in the debt-to-equity ratio leads to a rise in credit spreads.

[10] Specification (4) is similar to those used by Bharath and Shumway [2008] to predict credit default swap and corporate bond yield spreads. As in their paper, our main explanatory variables—volatility, expected profitability, and leverage—correspond to the "naïve" constituents of the distance-to-default, which, according to the Merton [1974] model, should be a sufficient statistic for default.

These results are robust to the inclusion of fixed credit rating effects (column 2) and to the inclusion of fixed industry effects (column 3). The specification in column 4 also controls for macroeconomic developments by including a full set of time dummies in the regression. Although the magnitude of the coefficient on uncertainty diminishes appreciably in this specification, the impact of uncertainty on credit spreads remains statistically significant and economically important: A 10 percentage point increase in uncertainty is associated with a rise in credit spreads of about 15 basis points. These results provide compelling evidence that fluctuations in uncertainty influence business financing conditions by significantly altering the level of credit spreads in the corporate bond market.

We now turn to the link between investment, uncertainty, and credit spreads. Our empirical investment equation is given by the following regression specification:

$$\log[I/K]_{it} = \beta_1 \log \sigma_{it} + \beta_2 \log s_{it} + \theta \log Z_{it} + \eta_i + \lambda_t + \epsilon_{it}, \tag{5}$$

where $[I/K]_{it}$ denotes the investment rate of firm i in period t (i.e., the ratio of capital expenditures in period t to the capital stock at beginning of the period); σ_{it} is our estimate of idiosyncratic uncertainty; s_{it} is the credit spread on the portfolio of bonds issued by firm i; and Z_{it} is a proxy for the marginal product of capital, a variable that measures firm i's future investment opportunities.[11] In addition to uncertainty, credit spreads, and investment fundamentals, the regression equation (5) includes a fixed firm effect η_i and a fixed time effect λ_t—the former controls for systematic differences in the average investment rate across firms, while the latter captures a common investment component reflecting macroeconomic factors, which can influence firm-level investment through either output or interest rates.[12]

We measure the investment fundamentals Z_{it} using either the current sales-to-capital ratio $[Y/K]_{it}$ or the operating-income-to-capital ratio $[\Pi/K]_{it}$. Taking logs of $[Y/K]_{it}$ is straightforward, but because operating income may be negative, we use $\log(c + [\Pi/K]_{it})$—where c is chosen so that $(c + [\Pi/K]_{it}) > 0$ for all i and t—when relying on the operating income to measure the firm's investment opportunities.[13] As an alternative forward-looking

[11] The frequency of data on capital expenditures and capital stock is annual, but the data are recorded at different months of the year, reflecting the differences in the fiscal years across firms. As a result, the uncertainty measure σ_{it} in equation (5) is calculated using daily idiosyncratic returns over the 250 trading days of the firm's fiscal year, and the credit spread is the average of the monthly credit spreads calculated over the 12 months of the firm's fiscal year. For the firms that have more than one bond issue trading in the secondary market in a given period, we calculate the portfolio spread by computing a weighted average of credit spreads on the firm's outstanding bonds, with weights equal to the market value of the issue.

[12] The log-log nature of regression (5) reflects the fact that the firm-level investment rates, uncertainty, and credit spreads are highly positively skewed, a feature of the data that is significantly ameliorated through the use of a logarithmic transformation.

[13] In principle, the estimated elasticities may depend on the constant c. In practice, however, reasonable variation in c has no effect on the estimated elasticities.

Table 2: Uncertainty, Credit Spreads, and Investment

Explanatory Variable	(1)	(2)	(3)	(4)	(5)	(6)
$\log \sigma_{it}$	-0.172	-0.086	-0.147	-0.060	0.008	-0.062
	(0.039)	(0.036)	(0.037)	(0.037)	(0.034)	(0.036)
$\log s_{it}$	-	-	-	-0.167	-0.149	-0.130
				(0.019)	(0.018)	(0.021)
$\log[Y/K]_{it}$	0.572	-	-	0.549	-	-
	(0.046)			(0.045)		
$\log[\Pi/K]_{it}$	-	1.292	-	-	1.207	-
		(0.081)			(0.081)	
$\log Q_{i,t-1}$	-	-	0.710	-	-	0.641
			(0.045)			(0.046)
R^2 (within)	0.289	0.260	0.251	0.309	0.277	0.262

NOTE: Sample period: firm-level monthly data from January 1973 to December 2009 at an annual frequency (No. of firms = 905; Obs. = 8,367). Dependent variable is $\log[I/K]_{it}$, the logarithm of the (real) investment rate of firm i in year t. All specifications include time fixed effects (not reported) and firm fixed effects, which are eliminated using the within transformation. The resulting specification is estimated by OLS. Heteroscedasticity- and autocorrelation-consistent asymptotic standard errors are clustered at the firm level and are reported in parentheses. Parameter estimates for $\log[\Pi/K]_{it}$ and the associated standard errors are adjusted for the fact that $\log[\Pi/K]_{it}$ is computed as $\log(0.5 + [\Pi/K]_{it})$.

measure of investment fundamentals, we also consider Tobin's Q, denoted by Q_{it}.

Result in columns 1–3 of Table 2 indicate a significant role for uncertainty in the investment process. Regardless of the measure of investment fundamentals, the coefficient on uncertainty is statistically highly significant and lies in the range between -0.17 and -0.09. These estimated elasticities of investment demand with respect to uncertainty imply that a 10 percentage point increase in uncertainty depresses the investment rate between one-half and three-quarters of a percentage point. However, once the credit spreads are included in the regression, columns 4–6, uncertainty ceases to be—either statistically or economically—an important determinant of investment spending. The coefficients on credit spreads, in contrast, are statistically highly significant and economically large, with a 100 basis points rise in credit spreads implying a drop in the investment rate of the same magnitude.

As a robustness check, we also considered a dynamic specification of the form:

$$\log[I/K]_{it} = \beta_1 \log \sigma_{it} + \beta_2 \log s_{it} + \theta_1 \log Z_{it} + \theta_2 \log[I/K]_{i,t-1} + \eta_i + \lambda_t + \epsilon_{it}.$$

In this case, we eliminated fixed firm effects using the forward orthogonal deviations transformation and estimated the resulting specification by GMM (cf. Arellano [2003]).[14] Al-

[14]The complete set of results is available from the authors upon request.

though the lagged investment rate had significant explanatory power, the results regarding the effect of both uncertainty and credit spreads on investment were virtually the same as those reported in Table 2. That is, the adverse effect of increased uncertainty on investment spending was completely attenuated once the information content of credit spreads was taken into account. In contrast, the impact of the change in this measure of financial frictions remained statistically and economically highly significant.

In summary, our aggregate time-series and firm-level panel analysis shows that the uncertainty-investment nexus is strongly influenced by conditions in the corporate bond market. In particular, increases in economic uncertainty are associated with a substantial widening of corporate credit spreads, which, in turn, leads to a significant contraction in economic activity. To the extent that credit spreads provide a useful barometer of the degree of frictions in the financial system, our empirical evidence indicates that financial frictions are an important conduit through which shocks in economic uncertainty are propagated to the real economy.

3 Structural Model

This section presents a general equilibrium model in which fluctuations in economic uncertainty influence bond prices and investment in a manner consistent with our empirical findings. The framework includes many of the salient features employed in the literature that allows for departures from the Modigliani and Miller [1958] paradigm of perfect capital markets, departures that imply a significant role for financial conditions in the determination of macroeconomic outcomes. In particular, firms use both internal and external sources of funds to finance capital expenditures. The presence of capital market imperfections implies that external funds command a premium and that this external finance premium increases in response to a rise in uncertainty.

3.1 Preferences, Technology, and Shocks

We consider a model with four types of economic agents: (i) a representative household; (ii) a continuum of firms producing final goods; (iii) a continuum of firms producing capital goods; and (iv) bond (i.e., financial) specialists. The representative household lives forever and maximizes the expected discounted sum of period-specific utilities $u(c, h)$, where c and h denote consumption of final goods and hours worked, respectively, and the utility function $u(\cdot, \cdot)$ is strictly increasing and concave in both arguments. The representative household earns a competitive real market wage w by working h hours and saves by purchasing equity shares of firms that produce final goods.

Firms in the final-goods sector combine capital and labor using a decreasing returns-

to-scale (DRS) Cobb-Douglas technology to produce output, which can be used for consumption or as an intermediate input in the production of new capital goods. The DRS production technology is subject to a persistent idiosyncratic productivity shock—denoted by z—that evolves according to

$$\log z' = \rho_z \log z + \log \epsilon'; \quad |\rho_z| < 1 \quad \text{and} \quad \log \epsilon' \sim N(-0.5\sigma^2, \sigma^2). \tag{6}$$

The assumptions underlying the production technology can be summarized by a function

$$y = z^{\nu} \left(k^{\alpha} h^{1-\alpha}\right)^{\gamma}, \tag{7}$$

where $0 < \alpha < 1$ is the value-added share of capital and $0 < \gamma < 1$ is the DRS parameter. The normalization parameter $\nu = 1 - (1 - \alpha)\gamma$ ensures that the firm's profit function $\pi(z, k) = z\pi(k)$ is linear in z.[15]

Because the producers of final goods employ a DRS technology, they earn strictly positive profits. To keep the model tractable, we do not explicitly model the firm's endogenous entry/exit decision. As in Cooley and Quadrini [2001], we assume that a constant fraction $0 < \eta < 1$ of final-goods producers exogenously exits the market in each period and that the same number of new firms enters the market within the same period. This stochastic overlapping generation structure also provides a convenient way to motivate the use of leverage by firms in the steady state without introducing a corporate income tax shield.

The capital-goods producers combine existing capital and final goods to produce new capital using a constant returns-to-scale (CRS) technology. The newly-produced capital is homogeneous and is sold at a competitive market price Q to the firms engaged in the production of final goods; the price Q denotes the price of capital goods relative to the price of final goods, the numeraire of the economy. Because of the CRS technology, the producers of capital goods earn zero profits in equilibrium, and that sector can be represented by a single firm. Bond investors provide debt financing to firms engaged in the production of final goods. A CRS technology is available to any bond investor, and the financial industry is assumed to be competitive. As a result, bond investors also earn zero profits in equilibrium.

To model time-varying economic uncertainty, we assume that the level of idiosyncratic uncertainty associated with the production technology in the final-goods industry evolves over time according to a persistent Markov process. Specifically, we assume that σ in

[15]The profit function can be derived from the following static optimization problem:

$$\max_{h} \{z^{\nu}(k^{\alpha}h^{1-\alpha})^{\gamma} - wh\}.$$

In contrast to the quasi-fixed nature of capital, labor hours are freely adjustable within a given time period, making the profit function convex in $\tilde{z} \equiv z^{\nu}$. The parameter ν then nullifies this convexity, making the profit function linear in z.

equation (6) follows a Markov Chain process with N states and a transition matrix $p(\sigma, \sigma')$. In our setup, a shock to the level of uncertainty corresponds to an aggregate shock that alters the level of uncertainty faced by all firms engaged in the production of final goods. Because ϵ' is distributed log-normally with $E(\epsilon'|\sigma) = \exp[0.5\sigma^2 + E(\log \epsilon'|\sigma)] = 1$, fluctuations in uncertainty do not change the conditional expectation of the productivity shock z; that is, an increase in uncertainty represents a mean-preserving spread to the conditional distribution of profits. As a result, fluctuations in uncertainty in our model do not have any direct implication for investment dynamics under the standard neoclassical assumptions.

With regards to timing, we assume that all economic agents in the model observe the realization of the idiosyncratic productivity shock z and the level of uncertainty σ at the beginning of each period. The timing convention is such that this period's uncertainty level σ determines the distribution of ϵ's in the subsequent period. An increase in uncertainty today, therefore, represents "news" to the economic agents regarding tomorrow's distribution of profits. To streamline notation, we let $s = (\sigma, \mu, K)'$ denote the vector of aggregate state variables, where μ denotes the joint distribution of idiosyncratic productivity shocks and net worth of the final-goods producers, and K is the aggregate stock of capital.

3.2 The Firm's Problem

To finance investment projects, firms producing final goods use a combination of internal and external funds, where the sources of external funds are debt and equity. Relative to internal funds, external funds command a premium, either because of the direct cost of issuing equity, or in the case of debt, because of the costs associated with default.

The net worth n of a firm engaged in the production of final goods is defined as

$$n \equiv z\pi(k, s) + Q(s)(1 - \delta)k - b, \tag{8}$$

where $Q(s)(1 - \delta)k$ is the resale value of installed capital k, and b is the face value of the bond issued by the firm in the previous period; $0 < \delta < 1$ denotes the depreciation rate of physical capital. Because we only consider one-period discount bonds, the market value of debt coincides with the face value of debt as long as the issuer does not default on its payment obligation.

The bond contract specifies the face value of the issue b' and the price q, yielding the total amount of debt financing qb'. Using these and other sources of funds, the firm purchases capital to be used in production. In the subsequent period—after observing the realization of shocks—the firm decides whether or not to fulfill its debt obligation. If the firm decides not to default, it pays the face value of the debt b' to the lender and makes its production and financial decisions for the next period. If the firm chooses to

default, it enters a debt-renegotiation process with the investor. The renegotiation process is conducted under limited liability by assuming that there exists a lower bound to the net worth of the firm—denoted by $\bar{n}$—below which the firm cannot promise to pay back any outstanding liability.[16]

Given the price of capital, its capital stock, and the amount of debt outstanding, the firm defaults if and only if the realized technology is lower than a threshold level $\bar{z}$, which is defined as the level that makes the firm's net worth equal to the default boundary:

$$\bar{n} = \bar{z}(k', b', s')\pi(k', s') + Q(s')(1-\delta)k' - b'. \tag{9}$$

Equation (9), in turn, defines a threshold level

$$\bar{\epsilon} \equiv \bar{\epsilon}(k', b', z, s') = \exp[\log \bar{z}(k', b', z, s') - \rho \log(z)],$$

such that the firm defaults if and only if $\epsilon' < \bar{\epsilon}$.

Under limited liability, the new level of debt renegotiated by the firm and the investor—denoted by b^R—cannot exceed the amount of debt $\bar{b}(k', z', s')$ that is consistent with the lower bound of the net worth:

$$b^R \leq \bar{b}(k', z', s') \equiv z'\pi(k', s') + Q(s')(1-\delta)k' - \bar{n}.$$

We assume that the firm does not have any bargaining power during the renegotiation process. Consequently, the renegotiated debt is set equal to the upper bound of the amount of debt that can be recovered—that is, $b^R = \bar{b}(k', z', s')$.

The default entails a dead-weight loss, captured by bankruptcy costs that are assumed to be proportional to the face value of the debt outstanding. Thus the actual recovery in the case of default is given by $b^R - \chi b'$, where the parameter $0 < \chi < 1$ governs the magnitude of the bankruptcy costs and hence the degree of frictions in the corporate bond market. Therefore, the recovery rate $\mathcal{R}$ in the case of default is given by

$$\mathcal{R}(k', b', z', s') = \frac{\bar{b}(k', z', s')}{b'} - \chi.$$

[16] This type of bond contract is similar to that of Merton [1974], Cooley and Quadrini [2001], and Hennessy and Whited [2007]. However, in our setup, a default occurs when the net worth of the firm n hits the lower bound $\bar{n}$, whereas in the aforementioned literature, a default occurs when the value of the firm V hits the lower bound $\bar{V}$. If the technology shock follows an i.i.d. process and the analysis is conducted in a partial equilibrium, the two assumptions are equivalent. However, if the technology shock is persistent or the firm's value function has other arguments, such as aggregate state variables, the two assumptions are no longer equivalent. The decision to use a lower bound for the net worth to determine the default threshold is a simplifying assumption that allows us to avoid the computationally intensive task of inverting the value function to compute the default boundary $\bar{n}(z, s)$ in each iteration of the dynamic programming routine.

The price of the bond is then equal to its discounted expected return:

$$q(k', b', z, s) = E\left[m(s, s')\left(1 + \int_{\epsilon' < \bar{\epsilon}} [\mathcal{R}(k', b', z', s') - 1] dH(\epsilon'|\sigma)\right)\Big| z, s\right], \tag{10}$$

where $m(s, s') = \beta u_c(s')/u_c(s)$ is the pricing kernel of the representative household, and $H(\cdot)$ denotes the CDF of the log-normal distribution. Using the properties of the log-normal distribution and letting $\bar{\theta}(k', b', z, s') = \frac{1}{\sigma}\left[\log \bar{\epsilon}(k', b', z, s') + \frac{\sigma^2}{2}\right]$, the price of the bond may be expressed as

$$\begin{aligned} q(k', b', z, s) \quad = \quad & E\Bigg[m(s, s')\Bigg(1 - \Phi\left(\bar{\theta}(k', b', z, s')\right) \\ & + \Phi\left(\bar{\theta}(k', b', z, s') - \sigma\right)\frac{z^{\rho}\pi(k', s')}{b'} \\ & + \Phi\left(\bar{\theta}(k', b', z, s')\right)\left(\frac{Q(s')(1-\delta)k' - \bar{n}}{b'} - \chi\right)\Bigg)\Big| z, s\Bigg]; \end{aligned}$$

where $\Phi(\cdot)$ denotes the standard normal CDF.

Because firms face a constant probability of exit η, the effective discount rate is equal to $(1-\eta)m(s, s')$, and risk-free rate $1/E[m(s, s')|s]$ is less than the inverse of the firm's discount factor $1/(1-\eta)E[m(s, s')|s]$. As a result, the firms are induced to hold a positive amount of debt in equilibrium. The exogenous exit shock occurs after the firm makes the payment decision on its existing debts (b), but before making its investment (k') and borrowing decision (b') for the current period. As a result, the exit shock does not directly affect the returns of bond investors.

At the margin, firms will only issue debt if equity issuance is also costly. We therefore posit the existence of a lower bound on dividends—denoted by $\bar{d}$—and a function governing the cost of issuing equity.[17] Specifically, the functional form of the per-unit cost of issuing equity is given by

$$\lambda(e) = \lambda_1 + \frac{\lambda_2}{2}e; \quad \lambda_1, \lambda_2 > 0,$$

where e is the amount of equity issued by the firm.

Given our setup, the firm's problem can be expressed recursively. Let d denote the firm's dividend:

$$d = z\pi(k, s) - Q(s)[k' - (1-\delta)k] - b + qb' + e. \tag{11}$$

[17] This modeling device is used by Cooley and Quadrini [2001], Hennessy and Whited [2007], and Jermann and Quadrini [2009]. More generally, costly equity issuance can be motivated by the existence of moral hazard or informational asymmetries between insiders and outside equity investors.

The value of the firm then solves the following dynamic programming problem:

$$\begin{aligned} V(n,z,s) &= \min_{\phi} \max_{k',b',d,e} \Big\{ d + \phi(d-\bar{d}) - [1+\lambda(e)]e \\ &\qquad + (1-\eta)E\Big[m(s,s')\max\{V(n',z',s'),V(\bar{n},z',s')\}\Big|z,s\Big]\Big\} \\ &\text{s.t.} \\ n' &= z'\pi(k',s') + Q(s')(1-\delta)k' - b', \end{aligned} \tag{12}$$

where ϕ is the Lagrange multiplier associated with the dividend constraint $d \geq \bar{d}$. The firm's continuation value is truncated by the default payoff and can be expressed as

$$E\Big[m(s,s')\max\Big\{V(n',z',s'),V(\bar{n},z',s')\Big\}\Big|z,s\Big] = \\ E\left[m(s,s')\left(\int_{\epsilon'<\bar{\epsilon}} V(\bar{n},z'(\epsilon'),s')d\Phi(\epsilon') + \int_{\epsilon'\geq\bar{\epsilon}} V(n',z'(\epsilon'),s')d\Phi(\epsilon')\right)\Big|z,s\right].$$

The first-order condition for equity issuance equates the shadow value of dividends to the marginal cost of issuance:

$$1+\phi = 1+\lambda(e)+\lambda'(e)e,$$

which implies that $\phi > 0$ when $e > 0$. In other words, it is never optimal for the firm to pay out more than the dividend bound $\bar{d}$ while issuing equity. Because equity financing is costly, a dollar of issuance reduces the value of existing shares more than a dollar, where the additional discount is given by $\lambda'(e)e$. The optimality of the firm's financial policy requires the firm to be indifferent between debt and equity finance. Accordingly, the first-order condition for debt issuance implies that

$$q(k',b',z,s) + q_b(k',b',z,s)b' = E\left[m(s,s')\int_{\epsilon'\geq\bar{\epsilon}}\left(\frac{1+\lambda(e')+\lambda'(e')e'}{1+\lambda(e)+\lambda'(e)e}\right)d\Phi(\epsilon')\Big|z,s\right], \tag{13}$$

where the term $q_b(k',b',z,s)b'$ captures the effect of increased leverage on borrowing costs.

The optimality conditions for capital accumulation imply the following Euler equation for investment:

$$\begin{aligned} Q(s) &= q_k(k',b',z,s)b' \\ &\quad + (1-\eta)E\left[m(s,s')\int_{\epsilon'\geq\bar{\epsilon}}\frac{1+\phi'}{1+\phi}[z'\pi_k(k',s') + (1-\delta)Q(s')]d\Phi(\epsilon')\Big|z,s\right]. \end{aligned} \tag{14}$$

This Euler equation has several non-neoclassical features. First, for any given level of borrowing b', an increase in capital raises the amount of available collateral and lowers

the threshold level of technology at which default occurs, effects captured by the term $q_k(k', b', z, s)b' > 0$. Second, the firm discounts the future cash-flows using the stochastic discount factor $(1 + \phi')/(1 + \phi)$, which is determined by the trade-off between debt and equity financing. Lastly, the expected marginal benefit of investment is truncated by the default boundary $\bar{\epsilon}(k', b', z, s')$, a consequence of introducing strategic default into the firm's optimization problem.

3.3 Market Clearing

Aggregate demand for capital is obtained by aggregating the individual demand functions:

$$I(s) = \int k'(n, z, s)di - (1 - \delta)K(s_{-1}),$$

where i indexes the continuum of firms in the final-good sector, and the argument n of function $k'(\cdot, \cdot, \cdot)$ pertains to the post-renegotiation value of the firm's net worth. Capital is supplied by a capital-goods producing sector that employs a CRS technology and takes the un-depreciated capital K and final goods I as inputs to produce new capital K'. The production of new capital is subject to adjustment costs $\xi(I(s)/K(s_{-1}))K(s_{-1})$, where the function $\xi(\cdot)$ is strictly convex. The new capital is sold to the producers of final goods at a unit price $Q(s)$, which is determined by the marginal cost of production:[18]

$$Q(s) = 1 + \xi'(I(s)/K(s_{-1})).$$

The efficiency conditions for the representative household can be summarized by a complete set of asset-pricing equations for the continuum of firms producing final goods and a first-order condition linking the marginal disutility of hours to the valuation of marginal consumption. The aggregate resource constraint is

$$\begin{aligned} C(s) = Y(s) - I(s) - \xi\left(I(s)/K(s_{-1})\right)K(s_{-1}) \\ - \int \Big[\mathbf{1}(n(i,s) \leq \bar{n})\chi b(i,s) + \mathbf{1}(e(i,s) \geq 0)\lambda(e(i,s))e(i,s)\Big]di, \end{aligned}$$

where $Y(s) = \int y(i)di$ and $\mathbf{1}(\cdot)$ denotes the indicator function that equals one if the argu-

[18] The optimization problem of the capital-goods sector—normalized by K given the CRS technology—can be formulated as

$$\max_{I(s)/K(s_{-1})} \left\{ Q(s)\left[\frac{I(s)}{K(s_{-1})} + (1 - \delta)\right] - \frac{I(s)}{K(s_{-1})} - Q(s)(1 - \delta) - \xi\left(\frac{I(s)}{K(s_{-1})}\right)\right\}.$$

Because of the capital adjustment costs, the value of existing capital depends separately on the joint distribution μ of net worth and technology, as well as on the current aggregate capital stock K, as indicated by the vector of aggregate state variables $s = (\sigma, \mu, K)'$.

ment is true and zero otherwise.[19] Compared with a frictionless real business cycle model, this constraint has two non-standard terms: the bankruptcy costs and equity issuance costs, which represent the loss of resources due to capital market imperfections. Because these costs are small relative to aggregate output, financial frictions modify the macroeconomic equilibrium primarily by altering the first-order conditions of the agents, rather than by directly affecting the available resources.

To fully solve the problem, economic agents need to understand how the aggregate state variables evolve over time. One of the aggregate state variables is the joint distribution of net worth and technology across heterogeneous firms. The exact law of motion for this joint distribution is given by

$$\begin{aligned}\mu(N_0, Z_0) = \int_{N_0 \times Z_0} \Bigg[\int_{N \times Z} \mathbf{1}\big(n' = \max\big\{\bar{n},\ [z'\pi(k'(n,z,s),s') \\ + Q(s')(1-\delta)k'(n,z,s) - b'(n,z,s)]\big\}\, G(z'|z,\sigma)d\mu\big) \Bigg] dn'dz',\end{aligned} \tag{15}$$

where $N \subseteq \mathbb{R}$, $Z \subseteq \mathbb{R}_{++}$, and μ is a measure on the measurable space $(N \times Z, \mathcal{N} \times \mathcal{Z})$, where $\mathcal{N}$ and $\mathcal{Z}$ denote Borel sigma algebras generated by the subsets of N and Z, respectively. Note that $\mu(N_0, Z_0)$ measures the proportion of firms with the net worth and technology in $N_0 \times Z_0$ next period, where $N_0 \in \mathcal{N}$ and $Z_0 \in \mathcal{Z}$. In equilibrium, this measure depends on (i) the firms' investment and debt policy functions $k'(n,z,s)$ and $b'(n,z,s)$; (ii) the transition function $G(z'|z,\sigma)$ of the idiosyncratic productivity shock z; and (iii) the aggregate market clearing conditions.[20]

Following the literature on computable general equilibrium with heterogeneous agents (cf. Krusell and Smith [1998]), we adopt the assumption of bounded rationality—that is, the agents concern themselves with only a finite number of moments of the distribution and use them in log-linear functional forms to forecast equilibrium prices. For computational purposes, agents in our model carry with them only the first moments of the distribution of net worth and technology as state variables. Agents use these state variables to forecast the three prices needed to solve their optimization problems: the marginal utility of the

[19] We assume that in the aggregate resource constraint there is no loss of output due to the exogenous exit of firms. That is, "death shocks" are realized after the firms produce output, and we assume that an entrant who replaces an exiting firm inherits all of its real and financial characteristics. The entry/exit process is thus fully frictionless and plays no role in the model, other than creating a wedge between the internal rate of discounting and the risk-free rate.

[20] Note that the distribution of net worth tomorrow also depends on the price of capital tomorrow—that is, $Q(s')$—because the collateral value of capital tomorrow depends on $Q(s')$. However, $Q(s')$ depends on the distribution of net worth and technology tomorrow, because the demand for capital tomorrow will depend on that distribution—hence the fixed point problem. Consequently, the aggregate law of motion in the firm's problem (12) is given by $\mu' = \Gamma(\mu, K, \sigma, \sigma')$, where we explicitly express the dependency of μ' on σ' and σ.

representative consumer ($u_c(s)$); real wage ($w(s)$); and the price of capital ($Q(s)$). The approximate laws of motion are given by the following system of linear regressions:

$$\log \mathbf{y} = \mathbf{C}(\sigma, \sigma_{-1}) + \mathbf{B} \log \mathbf{y}_{-1} + \mathbf{e}, \tag{16}$$

where the vector $\mathbf{y}$ includes the marginal utility of consumption $u_c(s)$, the aggregate net worth $N(s)$, and the aggregate capital stock $K'(s)$. The matrix of regression coefficients $\mathbf{B}$ is of the form

$$\mathbf{B} = \begin{bmatrix} 0 & b_{12} & b_{13} \\ 0 & b_{22} & b_{23} \\ 0 & b_{32} & b_{33} \end{bmatrix},$$

where the first column of zeros reflects the fact that the marginal utility of consumption is not a state variable. In the formulation of the aggregate laws of motion, we also allow the matrix of constants $\mathbf{C}$ in equation (16) to depend not only on the current realization of uncertainty, but also on its value in the previous period. Specifically, the system includes four distinct constant terms, corresponding to the four possible transitions for the uncertainty regime (i.e., "low-to-high," "low-to-low," etc.).[21]

4 Calibration

We let the time period t in our model correspond to one year—specifying the model at an annual frequency reduces computational time substantially. For the most part, our calibration relies on parameter values that are standard in the literature. However, there are a number of parameters that are specific to our model, the calibration of which we discuss below.

To calibrate the curvature of the profit function of firms engaged in the production of final goods and the parameters governing the stochastic uncertainty process, we utilize the S&P's Compustat (quarterly) database. Specifically, we selected from the Compustat database all U.S. nonfinancial firms with at least 20 quarters of data on sales and capital over the period 1976:Q1 to 2009:Q4, a procedure yielding an unbalanced panel of 9,469 firms for a total of 540,409 firm/quarter observations.[22] To calibrate γ, the DRS parameter

[21] Importantly, this allows us to obtain a much better goodness-of-fit for the approximate aggregate laws of motion relative to the specification that allows constant terms to differ only across the two uncertainty regimes. In the latter case, the goodness-of-fit statistics for the laws of motions—as measured by the R^2—were about 0.70 for most of endogenous aggregates. However, by allowing for regime switching, we obtained much better goodness-of-fit statistics: $R^2 = 0.951$ for the marginal utility of consumption and $R^2 = 0.998$ for the aggregate net worth. One exception was the law of motion for aggregate capital, where $R^2 = 0.910$ indicates a relatively poor fit.

[22] Prior to 1976, most firms did not report their capital stock data (i.e., net property, plant, and equipment) on the quarterly basis. To ensure that our results were not driven by a small number of extreme observations,

in equation (7), we use this panel to estimate the following revenue function:

$$\log Y_{it} = \beta \log K_{it} + \eta_i + \lambda_t + u_{it}, \tag{17}$$

where Y_{it} denotes (real) sales of firm i in quarter t, K_{it} is firm i's (real) capital stock at the beginning of the quarter, and the error term u_{it} represents the empirical counterpart of the productivity shock $\log z_t$ in our model. In our regression analysis, we include a firm fixed effect η_i to control for any unobservable (time-invariant) differences in the revenue process of individual firms, while the time fixed effect λ_t captures shocks affecting the profitability of all firms. Equation (17) is estimated by OLS yielding $\hat{\beta} = 0.618$, with the 95-percent confidence interval of $[0.606, 0.630]$. We calibrate α, the share of capital in the Cobb-Douglas production function (7) to be 0.30, which together with our estimate of β implies that $\gamma = 0.84$, an estimate of decreasing returns that is within the range of values estimated in the literature.

We use the residuals from the estimation of the revenue function (17) to calibrate the process for the idiosyncratic productivity shock. First, the persistence of the productivity process is obtained by estimating the following pooled regression

$$\hat{u}_{it} = \rho_z \hat{u}_{i,t-1} + \epsilon_{it},$$

which yields (at a quarterly frequency) $\hat{\rho}_z = 0.77$, implying the persistence of the process at an annual frequency of $0.77^4 = 0.35$; in our calibration, we set $\rho_z = 0.40$. Second, if ϵ_{it} is distributed normally, then $\sqrt{\pi/2}|\hat{\epsilon}_{it}|$ is an unbiased estimator of the standard deviation of ϵ_{it}.

To obtain a corresponding measure of time-varying uncertainty, we estimate the following panel regression:

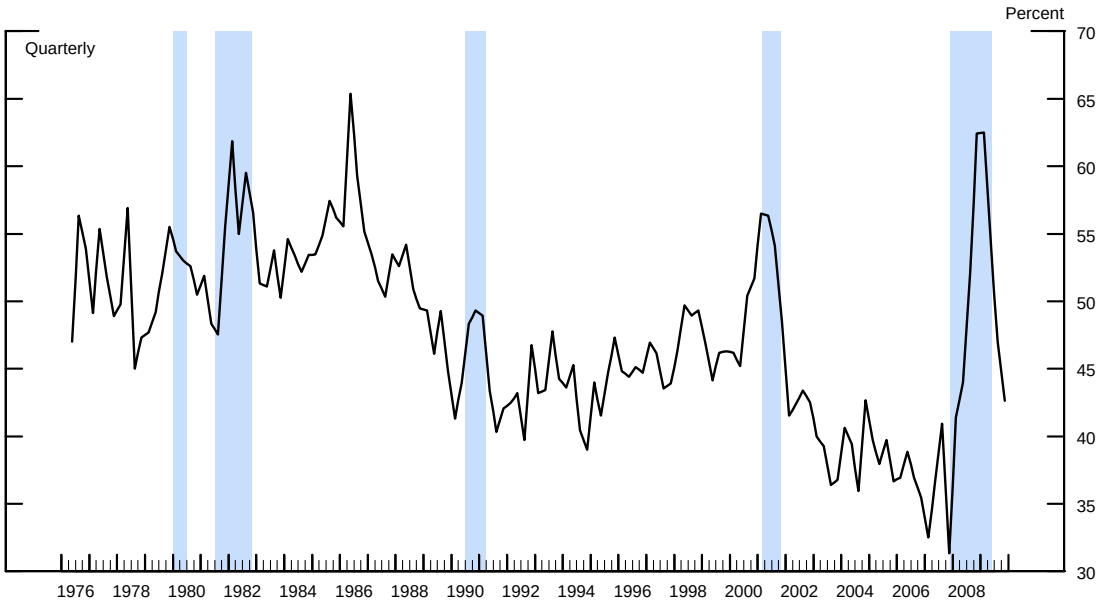
$$\log\left[\sqrt{\frac{\pi}{2}}|\hat{\epsilon}_{it}|\right] = \gamma_i + v_t + \zeta_{it}, \quad \zeta_{it} \sim N(0, \omega^2),$$

where γ_i and v_t denote fixed firm and time effects, respectively. In keeping with our earlier approach, a measure of uncertainty based on the shocks to the revenue function—shown in Figure 4—corresponds to the estimated sequence $\hat{v}_t$, $t = 1, \ldots, T$, which captures common movements in the idiosyncratic uncertainty regarding the profitability prospects in the non-financial corporate sector.[23] Note that like its counterpart based on equity valuations, this estimate of uncertainty is countercyclical, typically rising before an onset of an economic downturn.

we dropped from the sample all observations with the sales-to-capital ratio below 0.01 and above 20.0.

[23] To ease the interpretation, the estimates of fixed time effects v_t have been re-scaled, seasonally adjusted using the X11 filter, and expressed in annualized percent.

Figure 4: Uncertainty Based on Revenue Shocks



NOTE: Sample period: 1976:Q1–2009:Q4. The figure depicts an estimate of time-varying uncertainty based on shocks to the firm's revenue function (see text for details). The shaded vertical bars denote NBER-dated recessions.

In our simulations, the uncertainty process for σ is assumed to evolve according to a two-state Markov chain, with the two states corresponding to the "low" and "high" uncertainty regimes. To calibrate the Markov chain, we first estimate an AR(1) process for our measure of uncertainty based on the revenue shocks and then use the approach of Tauchen [1986] to discretize the process. Estimating $\hat{v}_t = \mu + \rho_\sigma \hat{v}_{t-1} + e_t$, yields an estimate of the autoregressive parameter $\rho_\sigma = 0.82$, with the 95 percent confidence interval of $[0.72, 0.92]$. We set the level of uncertainty corresponding to the low uncertainty regime—denoted by σ_L—to 35 percent and that in the high uncertainty regime—denoted by σ_H—to 55 percent; the steady-state level of dispersion $\bar{\sigma}$ is calibrated to 45 percent. The values for σ_L and σ_H correspond approximately to the 5th and 95th percentiles of the distribution of our uncertainty measures, whereas the value of $\bar{\sigma}$ is slightly below the median of the distribution. The probability that the uncertainty regime in period $t+1$ will be the same as in period t is set to 0.70, implying an AR(1) representation (at an annual frequency) with $\rho_\sigma = 0.82^4 = 0.45$.

We calibrate the degree of financial frictions in the bond market—the bankruptcy cost parameter χ—to match the median credit spread of 160 basis points for the 10-year BBB-Treasury spread over the 1976–2009 period. Accordingly, we set $\chi = 0.12$, a value consis-

tent with that used by Bernanke et al. [1999] and the micro-level evidence of Levin et al. [2004] and one that implies a relatively modest degree of welfare loss from bankruptcy. In calibrating the survival probability, we follow Carlstrom and Fuerst [1997] and let $1-\eta = 0.95^4 = 0.80$. The parametric form of per-unit cost of issuing equity $\lambda(e) = \lambda_1 + \frac{\lambda_2}{2}e$ implies that the marginal cost of issuing shares equals $1 + \lambda_1 + \lambda_2 e$. We set $\lambda_1 = 0.15$ and $\lambda_2 = 0.50$, values that generate a substantial price discount on newly issued equity and imply that equity is not a preferable source of external finance unless the firm is facing a substantial default-risk premium in the bond market.[24] This calibration generates a share of equity in total external financing of 11 percent, a proportion that is roughly in line with the average share of 8 percent reported by Bolton and Scharfstein [1996] for the U.S. corporate sector.

We consider two representations for the preferences of the representative household—a utility function that is separable in the marginal utilities of consumption and leisure and one that is not.[25] For our baseline case, we assume a log utility of consumption and linear disutility for hours: $u(c, 1-h) = \log c + \psi(1-h)$. In the non-separable case, we follow Greenwood et al. [1988] (GHH hereafter) and let $u(c, 1-h) = \log[c - (\psi/\theta)h^\theta]$, with the Frisch elasticity of labor supply $1/(\theta - 1) = 1.7$.[26] The household's subjective discounting factor is set equal to $0.99^4 = 0.96$, so that the annual risk-free rate is equal to 4 percent in the steady state.

The annual depreciation rate of physical capital δ is set to 18 percent, a value consistent with the firm-level Compustat data. We employ the following standard quadratic specification for the capital adjustment cost function: $\xi(I/K) = \frac{\vartheta}{2}(I/K - \delta)^2$. There is no clear consensus in the literature regarding the value of the adjustment cost parameter ϑ, with the range of published estimates running from 0.13 to 20.0. Early empirical work in particular has found a substantial degree of adjustment costs in the investment process,

[24] Our calibration of the cost of equity issuance falls in the range of empirical estimates found in the corporate finance literature. For example, the estimates of the cost for seasoned equity offerings range between 8 to 10 percent of the amount issued—where these costs include both the underwriting fee and the initial price discount—while the costs of private equity investment in public equity can be as high as 20 percent of the amount sold; see, for example, Hertzel and Smith [1993] and Wu [2004].

[25] The latter case is motivated by the fact that the driving force of economic fluctuations in our model works directly through the investment demand rather than through the resource constraint of the economy, as would be the case if the business cycles were due entirely to the fluctuations in TFP. In such an environment, the separability of marginal utility of consumption from that of leisure has important consequences for the comovement of key macroeconomic aggregates; see, for example, Barro and King [1984] and Greenwood et al. [1988]

[26] In the baseline case, the first-order condition with respect to hours worked is given by $w(s) = \psi/u_c(s) = \psi c(s)$, which implies that the labor supply function does not have to be approximated separately from the marginal utility of consumption in order to determine hours worked. As a result, $h(s)$ can be omitted from the vector $\mathbf{y}$ that describes the evolution of the aggregate state variables. In the case of the GHH preferences, in contrast, the first-order condition for hours worked is given by $h(s) = [\psi w(s)]^{1/(1-\theta)}$, which implies that the marginal utility of consumption and the labor supply function must be approximated separately.

while the more recent work indicates that this friction is likely to be less important. For example, using a large firm-level panel, Gilchrist and Himmelberg [1995] estimated ϑ to be around 3.0; using a simulation-based estimation method and plant-level data, Cooper and Haltiwanger [2006] estimated ϑ to be 0.13, when allowing for only convex adjustment costs in the capital adjustment process. In light of this evidence, we set $\vartheta = 1.0$.[27]

5 Simulation Results

5.1 Bond Pricing and Investment Policy

To examine the key features of our model, we first solve the model for the bond-pricing and investment policy functions by abstracting from the aggregate variation in idiosyncratic uncertainty. The top panel of Figure 5 shows the bond price (q) as a function of the firm's capital assets (k') and debt outstanding (b'), both of which are expressed relative to their steady-state values. The pricing surface has two distinct regions: A plateau in which the firm's leverage ratio b'/k' is sufficiently low so that the default probability is essentially zero and the price of debt is insensitive to the changes in the firm's financial condition; and a downward-sloping region, in which the firm faces increasing marginal cost of borrowing, and the price of debt drops sharply in response to an increase in leverage.

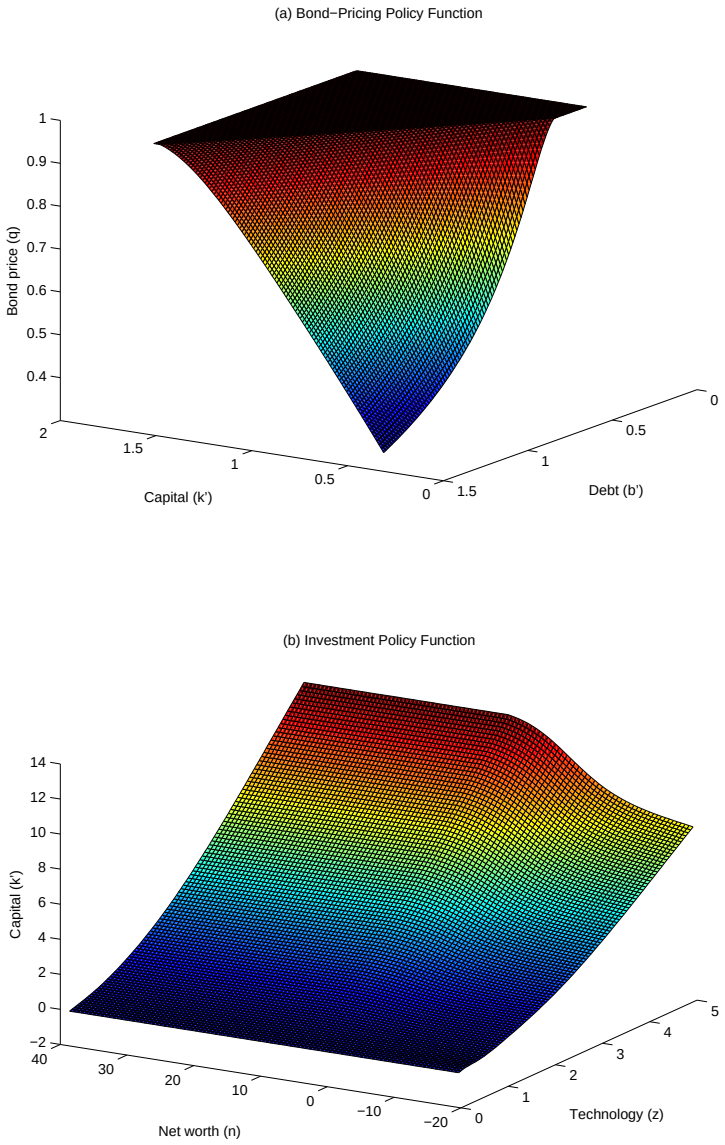
The bottom panel of the figure depicts the firm's optimal investment policy (k') as a function of net worth (n) and the technology level (z), with all variables again scaled by their respective steady-state levels. Like its optimal financial policy function, the firm's investment policy also exhibits a significant nonlinearity. In particular, at low levels of net worth, investment—for a given technology level—is highly responsive to the movements in the firm's net worth.

In Figure 6, we overlay the distribution of the debt-to-capital ratio of our model economy with the bond-pricing functions corresponding to the two uncertainty regimes.[28] When uncertainty is low, debt financing is relatively cheap and leverage is high. The overall distribution—across firms and time—of the debt-to-capital ratio is bimodal, reflecting the convolution of the distributions corresponding to the two uncertainty regimes. As shown in the figure, an increase in uncertainty affects aggregate investment by boosting the firms' borrowing costs, as evidenced by the downward shift in the bond-pricing function when the economy switches from a low to a high uncertainty regime. Faced with a significantly higher

[27] The model dynamics do not appear to be very sensitive to the value of ϑ, at least within the 0.13 to 3.0 range. A higher value of ϑ makes investment smoother by inducing a greater friction in the capital adjustment process; at the same time, higher capital adjustment costs make investment more volatile, because the value of collateral assets becomes more sensitive to cyclical fluctuations. The two effects tend to offset each other in our simulations.

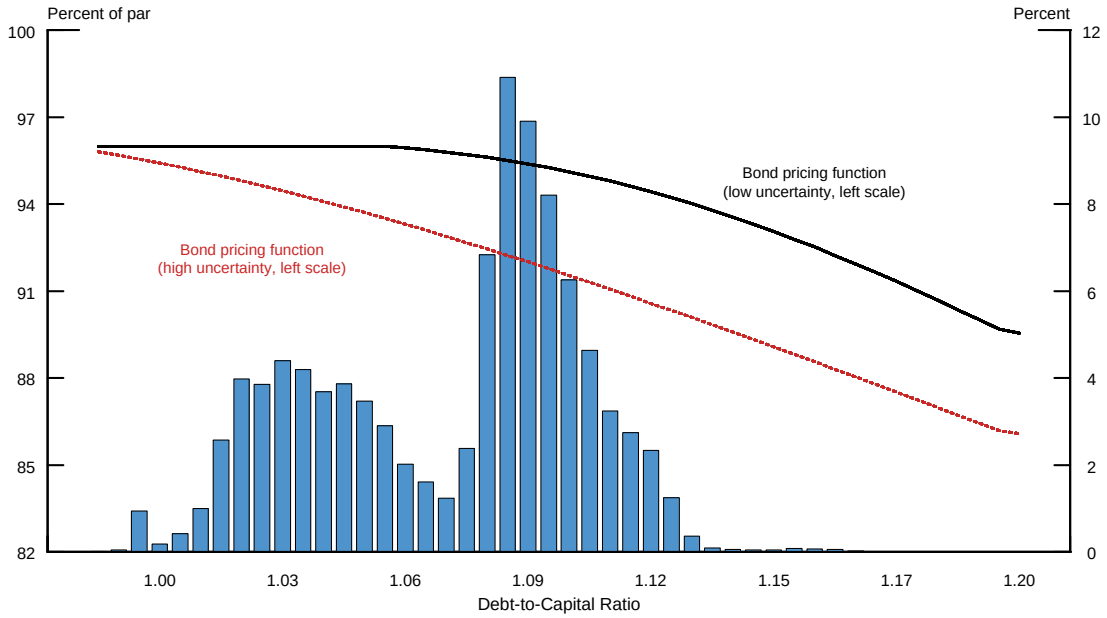
[28] The functions shown in Figure 6 are computed in a partial equilibrium setting, so that the risk-free rate is not affected by the change in the level of uncertainty.

Figure 5: Optimal Policy Functions



NOTE: Panel (a) depicts the bond price (q) as a function of the firm's capital assets (k') and debt outstanding (b). Panel (b) depicts the firm's optimal investment policy (k') as a function of net worth (n) and the level of technology (z). All variables are expressed relative to their respective steady-state values. The policy functions are computed under the assumption of no aggregate shock to idiosyncratic uncertainty (see text for details).

Figure 6: Uncertainty and the Bond-Pricing Policy Function



NOTE: The histogram depicts the model-implied distribution—across firms and time—of the debt-to-capital ratio (b/k). The solid line depicts the optimal bond-pricing policy as a function of the debt-to-capital ratio in the low uncertainty regime ($\sigma_L = 0.35$), and the dotted line depicts the bond-pricing policy function in the high uncertainty regime ($\sigma_H = 0.55$).

cost of debt finance, firms in the model simultaneously deleverage and cut back on capital expenditures.

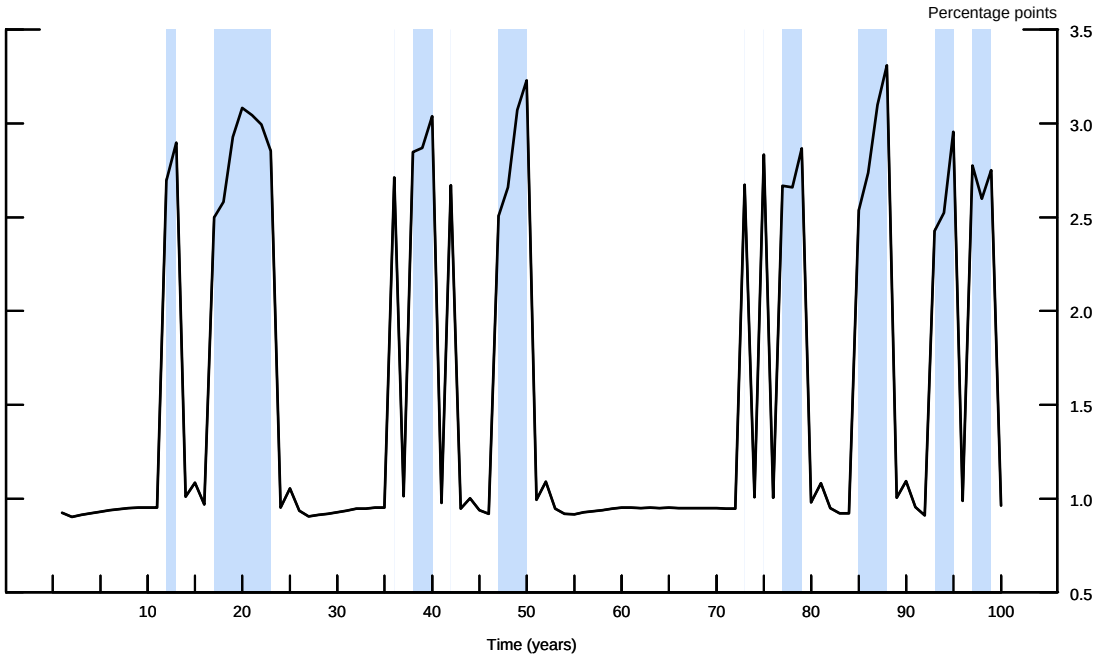
5.2 Uncertainty, Credit Spreads, and Economic Fluctuations

We now report our main simulation results.[29] The simulation is designed so that the average level of technology in the economy is constant, while the dispersion of technology fluctuates over time according to the two-state Markov chain process. The solid line in Figure 7 shows the the model-implied credit spread computed under the risk-neutral measure, using the baseline specification of the preferences of the representative household (i.e., log utility of consumption and linear disutility of hours worked). The shaded vertical bars indicate periods in which the economy is in the high uncertainty regime.

According to the figure, periods of heightened uncertainty are associated with elevated credit spreads. In the transition from the state of low uncertainty to that of high uncertainty,

[29]The model is simulated for 400 periods (i.e., years), assuming that there are always 10,000 firms in the economy. All the figures show the simulated time path of the model's key financial and macroeconomic variables for 100 "years" of data, corresponding to the period from t_{151} to t_{250}. All the statistics based on the simulated data are computed using the full sample of 400 observations.

Figure 7: Uncertainty and Credit Spreads



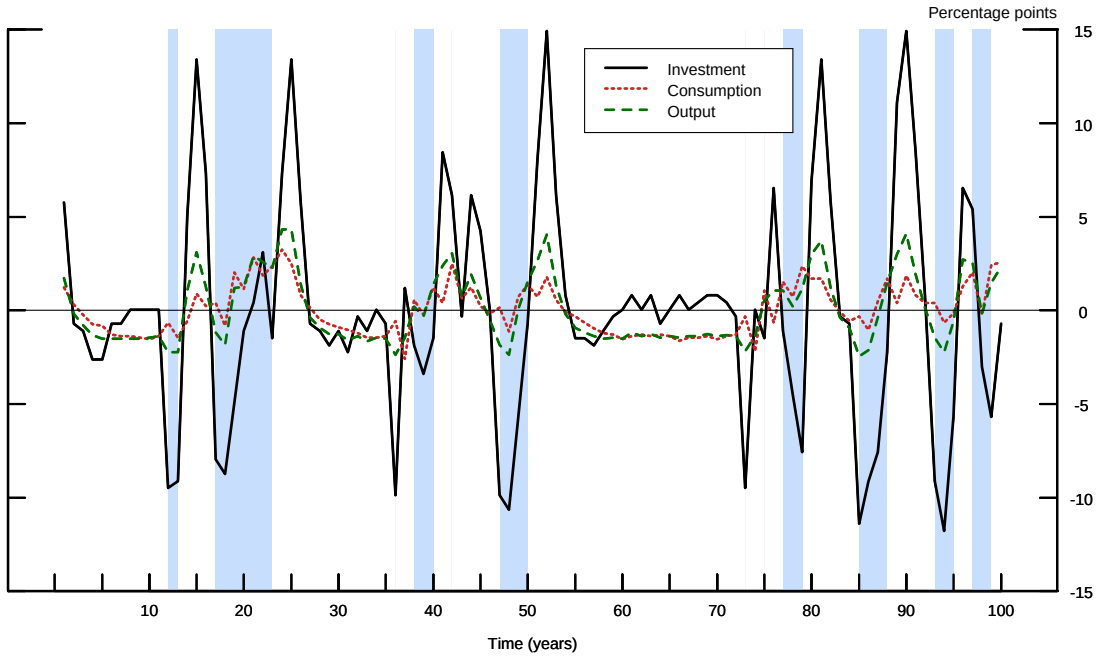
NOTE: The figure shows the simulated time path of the average credit spread based on bond prices computed under the risk-neutral measure, using the baseline specification of the preferences of the representative household (see text for details). The shaded vertical bars correspond to periods of high uncertainty.

credit spreads jump about 150 basis points and then tend to increase another 60 basis points or so as the high uncertainty regime persists. This further worsening of credit conditions reflects the endogenous interaction between the real and financial sides of the economy. In particular, the transition to the state of high uncertainty depresses investment because the resulting increase in the downside risk leads to a widening of credit spreads. The initial drop in investment has adverse implications for both current asset prices and future profits, which raises the likelihood of subsequent defaults and causes credit spreads to widen further—the financial accelerator mechanism.

Figure 8 displays the evolution of the key macroeconomic aggregates—expressed in deviations from their steady-state values—in the baseline specification of our model. As shown by the black line, the transition from a low to a high uncertainty regime is associated with an immediate drop in aggregate investment; conversely, the transition from a high to a low uncertainty regime generates an investment boom. The size of these fluctuations is quite substantial, ranging between 10 to 15 percent of the steady-state level of investment.

Figure 8 also shows that our model implies a high degree of comovement between ag-

Figure 8: Uncertainty and Real Economic Activity



NOTE: The figure shows the simulated time path of aggregate investment (solid line), consumption (dotted line), and output (dashed line), using the baseline specification of the preferences of the representative household (see text for details). All three series are expressed in percentage-point deviations from their steady-state values. The shaded vertical bars correspond to periods of high uncertainty.

gregate consumption, investment, and output. In Table 3, we compare some standard business cycle statistics to their model-implied counterparts. Recall that the baseline case corresponds to the model with the log utility of consumption and linear disutility of hours worked, whereas the GHH model denotes the specification with non-separable preferences. The top panel of the table shows that, with the exception of personal consumption expenditures, our model successfully replicates the relative volatilities of the key macroeconomic aggregates. In particular, in both specifications, the model-implied investment is two to three times more variable than output, a result that accords well with the U.S. historical experience. In the baseline model, the relative volatility of the model-implied hours is very close to that observed in the actual data, while the relative volatility of hours worked in the model with GHH preferences is too low, reflecting in part the excessive variability of output.

The bottom panel of the table summarizes the comovement properties of the model. As shown in the middle column, our baseline case implies a strong comovement among the main endogenous quantities. With the exception of hours worked, the correlation coeffi-

Table 3: Descriptive Business Cycle Statistics

	Relative Standard Deviation[a]		
Variable	Actual Data	Baseline Model	GHH Model
Consumption	0.541	0.704	0.863
Investment	3.343	3.315	2.207
Hours	1.217	1.120	0.671
Measured TFP	1.363	0.861	0.571
Memo: $STD(Y)$	0.014	0.019	0.025
	Correlation with Output		
Variable	Actual Data	Baseline Model	GHH Model
Consumption	0.875	0.859	0.950
Investment	0.833	0.771	0.746
Hours	0.900	0.814	0.989
Measured TFP	0.819	0.811	0.815
Memo: $Corr(C, I)$	0.736	0.375	0.526

NOTE: Sample period for the actual annual data: 1954–2009 ($T = 56$). Actual data are in logs and have been detrended using the Hodrick-Prescott filter with $\lambda = 6.25$; see Ravn and Uhlig [2002] for details.

[a]Scaled by the standard deviation of detrended output.

cients based on the simulated data from the baseline economy closely match their empirical counterparts, a rather remarkable result given that all the comovements are generated in the absence of shocks to the technological frontier. Although the level of technology is not changing in response to the fluctuations in uncertainty, resources are nevertheless being reallocated from high productivity firms with low net worth to low productivity firms with high net worth because of the constraints imposed on capital formation by the increased severity of financial frictions. As a result, the model implies a positive correlation between measured TFP and output fluctuations that is also very close to the one found in the U.S. data.[30]

Unlike technology shocks, which affect the agents' behavior by altering the supply side of the economy, uncertainty shocks also affect the cost of investment relative to consumption and thus are akin to the "investment efficiency shocks" of Greenwood et al. [1988]. A shock that increases the rate of return on investment will also cause a temporary decline in consumption and an increase in labor supply through the intertemporal substitution of leisure.

[30]For this comparison, we constructed a TFP series for both the data and the model economies using the conventional formula: TFP $= \exp[\log(Y) - 0.3\log(K) - 0.7\log(L)]$. Correcting for the effects of a DRS technology matters very little for our conclusion.

Table 4: Cyclical Properties of Aggregate Investment

	Correlation with Investment Growth		
Variable	Actual Data	Baseline Model	GHH Model
Credit spread	-0.597	-0.536	-0.345
Net worth growth	0.390	0.498	0.417
Uncertainty	-0.360	-0.592	-0.360

NOTE: Sample period for the actual quarterly data: 1963:Q4–2009:Q4 ($T = 185$). Entries for the actual data are the correlations between the log-difference of real business fixed investment, the level of the 10-year BBB-Treasury spread, the log-difference of real net worth for the nonfinancial corporate sector, and our benchmark estimate of time-varying uncertainty.

The productivity benefits associated with capital reallocation strengthen the wealth effect, which in the baseline model implies procyclical consumption and countercyclical hours. In contrast, owing to the strong complementarity, the model with GHH preferences produces a positive comovement between consumption and hours. As a result, the correlation between output and hours is positive and of the same order of magnitude as that seen in the data. In addition, the correlation coefficient between consumption and investment accords much better with its empirical counterpart.

The results in Table 4 indicate that our model also captures quite well the cyclical comovements between investment, the key financial variables—credit spreads and borrowers' net worth—and uncertainty. In particular, both model specifications deliver a strong negative correlation between investment growth and credit spreads, though the magnitude of this negative comovement accords somewhat better with the actual data in the baseline case. In contrast, the model with GHH preferences is able to match much more closely the observed positive correlation between the changes in the net worth of the U.S. nonfinancial corporate sector and the growth rate of business fixed investment.[31] In addition, the correlation between uncertainty and investment growth in the specification with the GHH preferences is essentially identical to the one found in the data.

5.3 Capital Reallocation

As shown by Eisfeldt and Rampini [2006], the benefits of capital reallocation increase during economic downturns, while the actual amount of capital reallocation declines during recessions. In our model, the benefits of capital reallocation are also countercyclical, because the

[31]The net worth data come from the Federal Reserve's Z.1 statistical releases, "Flow of Funds Accounts of the United States." The nominal net worth series was deflated by the implicit GDP price deflator.

Table 5: Cyclical Properties of Capital Reallocation

Variable	Baseline Model	GHH Model
Output	0.752	0.814
Consumption	0.782	0.796
Investment	0.370	0.482
Measured TFP	0.752	0.903
Memo: RAC/Investment	0.317	0.317

NOTE: Entries in the table denote the correlation coefficients between RAC—our measure of capital reallocation—and the specified variable, measured in percentage-point deviations from their steady-state values. Capital reallocation is defined as the sum of all investment flows associated with the redeployment of the existing capital stock (see text for details).

increase in dispersion of productivity shocks is the cause of the economic downturn. Capital reallocation (RAC) can be measured as the difference between gross and net investment flows:

$$\text{RAC}_t = \sum_n w_{nt} \left(|i_{nt}| - i_{nt} \right), \tag{18}$$

where i_{nt} is the investment of firm n in period t and $w_{nt} = k_{nt} / \sum k_{nt}$ is the corresponding weight. In effect, equation (18) measures reallocation as the amount of capital that changes ownership across firms.

Table 5 examines the cyclical properties of this reallocation measure for both the baseline and GHH model specifications. According to the memo item in the table, the amount of capital reallocation—measured as a proportion of the steady-state level of gross investment—is substantial in both model specifications and very close to that calculated by Eisfeldt and Rampini [2006] using the firm-level Compustat data. Moreover, capital reallocation in both specifications is strongly procyclical, as evidenced by the positive correlation of RAC with the key economic aggregates.[32] More importantly, the amount of capital reallocation moves closely with the fluctuations in TFP, with the correlation coefficients in the range of 0.75 to 0.90. A similar degree of comovement was obtained by Eisfeldt and Rampini [2006] by assuming countercyclical capital adjustment costs. Our setup, in contrast, uses financial market frictions to effectively endogenize the countercyclical nature of capital adjustment

[32]The correlation between RAC and output shown in the table is somewhat higher than that reported by Eisfeldt and Rampini [2006]. This difference, however, seems natural—in our model, there is only one shock that affects the dispersion of productivity. As emphasized by Eisfeldt and Rampini [2006], new investment is driven by the changes in aggregate productivity, while the amount of capital reallocation is influenced mainly by the differences in productivity across firms. In reality, however, there are likely multiple shocks that affect macroeconomic outcomes, with some of these shocks having a direct impact on the aggregate productivity, thereby dampening the degree of comovement seen in the data.

Table 6: Uncertainty and the Corporate Bond Market

Uncertainty, Credit Spreads, and the Risk Premium			
Credit Spread (bps.)	Actual Data[a]	Baseline-PHM	Baseline-RNM
Unconditional	161	164	174
Conditional on $\sigma = \sigma_L$	-	94	96
Conditional on $\sigma = \sigma_H$	-	256	277
Uncertainty and Default Rates			
Default Rate (pct.)	Actual Data[b]	Baseline-PHM	Baseline-RNM
Unconditional	0.38	2.37	2.40
Conditional on $\sigma = \sigma_L$	-	1.88	1.88
Conditional on $\sigma = \sigma_H$	-	3.00	3.04

NOTE: PHM = physical measure; RNM = risk-neutral measure; $\sigma = \sigma_L$ corresponds to the low uncertainty regime; and $\sigma = \sigma_H$ corresponds to the high uncertainty regime.

[a]Average 10-year BBB-Treasury credit spread (1976:Q1–2009:Q4).

[b]Average quarterly U.S. nonfinancial bond default rate (1981:Q1–2009:Q4).

costs—the intensification of financial frictions during an economic downturn limits the firm's investment relative to its fundamentals as measured by the changes in productivity.[33]

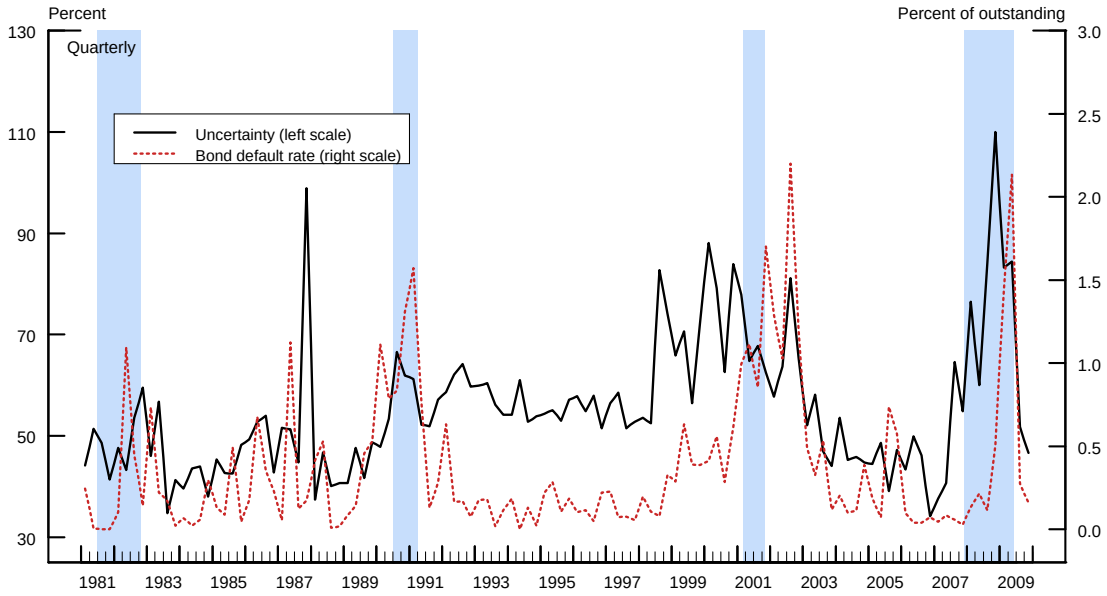
5.4 Uncertainty and the Risk Premium

As documented in the corporate finance literature (Elton et al. [2001]), traditional debt-contracting models imply counterfactually low credit spreads—and hence significant risk premiums—given the observed probabilities of default and actual recovery rates. In this section, we examine the behavior of the bond risk premium by decomposing the average credit spread implied by our baseline model into two components: the credit spread based on the physical measure and the risk premium.

The results of this exercise, which are shown in the top panel of Table 6, indicate only a modest role for the risk premium as a determinant of yield spreads on corporate debt. The unconditional risk premium in our baseline model—the difference between the average credit spread computed under the risk-neutral measure (RNM) and that under the physical measure (PHM)—is only 10 basis points. In addition to accounting, on average, for only small portion of credit spreads, the risk premium is present only during regimes of high uncertainty—conditional on the economy being in the high uncertainty regime, the

[33]Khan and Thomas [2008] reach a similar conclusion in a dynamic general equilibrium setup with borrowing constraints and a partial investment irreversibility.

Figure 9: Uncertainty and Corporate Bond Defaults



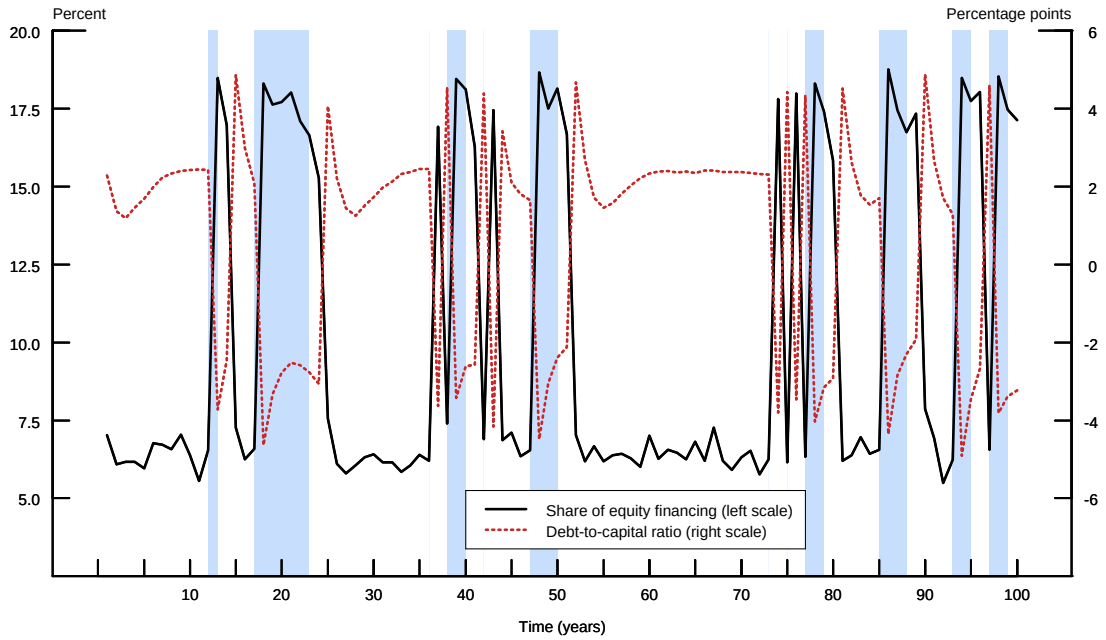
NOTE: Sample period: 1981:Q4–2009:Q4. The solid line depicts our benchmark estimate of time-varying uncertainty based on equity valuations (see text for details). The dotted line depicts the nonfinancial bond default rate in quarter t, calculated as the sum of defaults during the quarter, divided by the amount outstanding at the beginning of the quarter. The shaded vertical bars represent the NBER-dated recessions.

premium is about 20 basis points, while it is essentially zero in the low uncertainty regime. Although the risk premium is clearly countercyclical, the model is unable to generate a premium that can systematically account for a significant portion of credit spreads.

Because our baseline specification of the model is calibrated to match the average level of credit spreads, an implication of these findings is that the model-implied default rates exceed those found in the data by a considerable margin. According to the available evidence, the unconditional model-implied default rates are, on average, about 2.0 percentage points higher than the actual average default rate on nonfinancial corporate bonds (bottom panel of Table 6). The inability of our model to match more closely the observed average default rate is due in part to the more general failure of the expected utility theory to provide empirically realistic risk premiums. Although the average excess return on both stocks and corporate bonds in our model is strongly procyclical, the average risk premium in both markets is substantially lower than their respective empirical counterparts.[34]

[34]In an attempt to resolve this type of pricing anomalies, Gomes and Schmid [2009] incorporate the Epstein-Zin preferences into a dynamic general equilibrium setting with costly state verification, while Chen et al. [2009] embed the standard Merton framework into a partial equilibrium framework with habit formation. Both approaches make some important progress in helping to resolve the "credit spread" puzzle.

Figure 10: Uncertainty and External Financing Patterns



NOTE: The solid line shows the model-implied share of equity financing, and the dotted line depicts the debt-to-capital ratio, expressed in percentage-point deviations from its steady-state value. The shaded vertical bars correspond to periods of high uncertainty.

While the model is unable to deliver empirically realistic default rates, it does predict that defaults should rise in periods of heightened uncertainty. According to Figure 9, this is indeed the case—our benchmark estimate of uncertainty is highly positively correlated with the realized bond defaults. In addition, the average actual bond default rate during the last three recessions is considerably closer to the average model-implied default rate, conditional on the economy being in the high uncertainty regime.

5.5 External Financing Patterns

As shown by the solid line in Figure 10, firms seek to avoid the increased cost of bond finance associated with the elevated uncertainty by issuing equity and deleveraging their balance sheets. The extent of this substitution, however, is limited. In our model economy, equity, on average, accounts for about 11 percent of all external funds. Despite the sharp

However, Gomes and Schmid [2009] allow for only the extensive margin of investment and abstract from the endogenous labor supply decision, an approach that makes it difficult to analyze the business cycle implications of their model. Chen et al. [2009] show that it is necessary to introduce a procyclical default boundary—essentially a shorter distance-to-default in recessions—to generate a sufficiently large risk premium component of corporate credit spreads.

increase in volume, the share of equity financing rises only about 10 percentage points in the high uncertainty regimes. In effect, rising marginal cost limit the amount of equity issued.

According to Figure 10, the model-implied share of equity finance is countercyclical, while the leverage is procyclical. Although the latter result is consistent with the data, the model-implied pattern of equity financing runs contrary to the observed cyclical behavior of equity issuance.[35] This counterfactual result likely reflects the reduced-form nature of the costs governing equity issuance. In particular, a more realistic description of external financing patterns would recognize that a firm issuing equity in an environment of elevated economic uncertainty may face moral hazard or asymmetric information problems similar to those encountered when trying to place debt with the bond investors.

6 Conclusion

According to the standard macroeconomic theory, investment irreversibilities are the main channel through which fluctuations in uncertainty affect capital formation. In this paper, we exploit the implications of uncertainty for the cost of external debt finance by developing a general equilibrium framework in which financial market frictions provide the link between uncertainty and the aggregate investment cycle. The notion that conditions in the financial markets are an important conduit through which fluctuations in uncertainty are transmitted through to the real economy is strongly supported by our empirical evidence. According to both the macro and micro data, increases in uncertainty lead to the widening of spreads on corporate bonds and protracted declines in investment and output.

The quantitative general equilibrium structure of our model implies that empirically realistic increases in uncertainty can replicate the negative comovement between credit spreads and investment, the positive comovement between net worth and investment, while also accounting for many of the salient characteristics of the business cycle fluctuations. By allowing for heterogeneity in productivity and net worth across firms, the model also implies an important reallocation mechanism for the economy as a whole, a mechanism that generates a procyclical reallocation of factor inputs and, as a result, procyclical movements in measured TFP. Overall, our simulations demonstrate that fluctuations in economic uncertainty have important consequences for macroeconomic outcomes in an environment that allows for the departures from the Modigliani-Miller paradigm of frictionless financial markets.

[35] Choe et al. [1993] and Bayless and Chaplinsky [1996] provide microeconomic evidence regarding the procyclical nature of equity financing. In contrast, Jermann and Quadrini [2009] show that aggregate equity issuance is countercyclical. Covas and Den Haan [2007] show that this dichotomy reflects the disproportionate influence of very large firms.

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Appendices

A Data Sources and Methods

The key information underlying the firm-level analysis comes from a large sample of fixed income securities issued by U.S. nonfinancial corporations. Specifically, from the Lehman/Warga (LW) and Merrill Lynch (ML) databases, we obtained month-end prices of outstanding long-term corporate bonds that are actively traded in the secondary market.[36] To guarantee that we are measuring borrowing costs of different firms at the same point in their capital structure, we restricted our sample to senior unsecured issues with a fixed coupon schedule only. For such securities, we spliced their month-end prices across the two data sources.

The micro-level aspect of our data set allows us to construct credit spreads that are not subject to the maturity/duration bias. In particular, we construct for each individual bond issue a theoretical risk-free security that replicates exactly the promised cash-flows of the corresponding corporate debt instrument. For example, consider a corporate bond k issued by firm i that at time t is promising a sequence of cash-flows $\{C(s) : s = 1, 2, \ldots, S\}$, consisting of the regular coupon payments and the repayment of the principle at maturity. The price of this bond in period t is given by

$$P_{it}[k] = \sum_{s=1}^{S} C(s)D(t_s),$$

where $D(t) = e^{-r_t t}$ is the discount function in period t. To calculate the price of a corresponding risk-free security—denoted by $P_t^f[k]$—we discount the promised cash-flow sequence $\{C(s) : s = 1, 2, \ldots, S\}$ using continuously-compounded zero-coupon Treasury yields in period t, obtained from the daily estimates of the U.S. Treasury yield curve reported by Gürkaynak et al. [2007]. The resulting price $P_t^f[k]$ can then be used to calculate the yield—denoted by $y_t^f[k]$—of a hypothetical Treasury security with exactly the same cash-flows as the underlying corporate bond. The credit spread $s_{it}[k] = y_{it}[k] - y_t^f[k]$, where $y_{it}[k]$ denotes the yield of the corporate bond k, is thus free of the "duration mismatch" that would occur were the spreads computed simply by matching the corporate yield to the estimated yield of a zero-coupon Treasury security of the same maturity.

To ensure that our results are not driven by a small number of extreme observations, we eliminated all bond/month observations with credit spreads below 5 basis points and with spreads greater than 3,500 basis points. In addition, we dropped from our sample very small corporate issues—those with a par value of less than $1 million—and all observations with a

[36] These two data sources are used to construct benchmark corporate bond indexes used by the market participants. Specifically, they contain secondary market prices for a significant fraction of dollar-denominated bonds publicly issued in the U.S. corporate cash market. The ML database is a proprietary data source of daily bond prices that starts in 1997. Focused on the most liquid securities in the secondary market, bonds in the ML database must have a remaining term-to-maturity of at least two years, a fixed coupon schedule, and a minimum amount outstanding of $100 million for below investment-grade and $150 million for investment-grade issuers. By contrast, the LW database of month-end bond prices has a somewhat broader coverage and is available from 1973 through mid-1998 (see Warga [1991] for details).

remaining term-to-maturity of less than one year or more than 30 years; calculating spreads for maturities of less than one year and more than 30 years would involve extrapolating the Treasury yield curve beyond its support.[37] These selection criteria yielded a sample of 5,378 individual securities between January 1973 and December 2009. We matched these corporate securities with their issuer's quarterly and annual income and balance sheet data from Compustat and daily data on equity valuations from CRSP, yielding a matched sample of 944 firms.

Table A-1 contains summary statistics for the key characteristics of bonds in our sample. Note that a typical firm has only a few senior unsecured issues outstanding at any point in time—the median firm, for example, has two such issues trading at any given month. This distribution, however, exhibits a significant positive skew, as some firms can have as many as 74 different senior unsecured bond issues trading in the market at a point in time. The distribution of the real market values of these issues is similarly skewed, with the range running from $1.2 million to more than $5.6 billion. Not surprisingly, the maturity of these debt instruments is fairly long, with the average maturity at issue of about 13 years. Because corporate bonds typically generate significant cash flow in the form of regular coupon payments, the effective duration is considerably shorter, with both the average and the median duration of about 6 years.

According to the S&P credit ratings, our sample spans the entire spectrum of credit quality, from "single D"to "triple A." At "BBB1," however, the median bond/month observation is still solidly in the investment-grade category. Turning to returns, the (nominal) coupon rate on these bonds averaged 7.31 percent during our sample period, while the average total nominal return, as measured by the nominal effective yield, was 7.82 percent per annum. Reflecting the wide range of credit quality, the distribution of nominal yields is quite wide, with the minimum of 0.66 percent and the maximum of more than 44 percent. Relative to Treasuries, an average bond in our sample generated a return of about 202 basis points above the comparable risk-free rate, with the standard deviation of 284 basis points.

[37] We also eliminated a small number of putable bonds from our sample.

Table A-1: Summary Statistics of Corporate Bond Characteristics

Bond Characteristic	Mean	SD	Min	P50	Max
# of bonds per firm/month	2.83	3.46	1.00	2.00	74.0
Mkt. value of issue[a] ($mil.)	310.1	315.6	1.22	231.0	5,628
Maturity at issue (years)	13.3	9.5	1.0	10.0	50.0
Term to maturity (years)	11.4	8.6	1.0	8.2	30.0
Duration (years)	6.50	3.20	0.91	6.10	15.6
Credit rating (S&P)	-	-	D	BBB1	AAA
Coupon rate (pct.)	7.31	1.95	1.95	7.00	17.5
Nominal effective yield (pct.)	7.82	3.24	0.66	7.25	44.3
Credit spread (bps.)	202	284	5	116	3,499

Panel Dimensions

Obs. $= 345,785$ $N = 5,378$ bonds

Min. Tenure $= 1$ Median Tenure $= 53$ Max. Tenure $= 327$

NOTE: Sample period: Monthly bond-level data from January 1973 to December 2009 for a sample of 944 nonfinancial firms. Sample statistics are based on trimmed data (see text for details).

[a]Market value of the outstanding issue deflated by the CPI (1982–84 = 100).